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Data Analysis II: periodic sources

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http://grwavsf.roma1.infn.it/pss/basic

Data Analysis Document: http://grwavsf.roma1.infn.it/dadps/

Outline

- Signal characterization
- Basic detection techniques and b.d.t. computational load
- Hough (and Radon) transform
- Hierarchical search
- Coherent step
- Detection policy
- Pulsar spectroscopy
- Dither effect

Peculiarity of the periodic sources

The periodic sources are the only type of gravitational signal that can be detected by a single gravitational antenna with certainty (if there is enough sensitivity to include the source among the candidates, the **false alarm probability** can be reduced at any level of practical interest).

The estimation of the source parameters (like e.g. the celestial coordinates) can be done with the highest precision.

Once one detects a periodic source, it remains there to be confirmed and studied by others. It is not only a detection, it is a discover.

Signal characterization

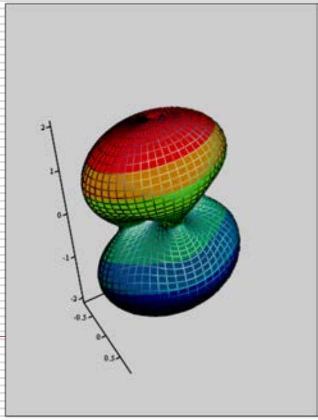
- **Shape:** sinusoidal, possibly two harmonics.
- **Location:** our galaxy, more probable near the center or in globular clusters; nearest (and more detectable) sources are isotropic; **sometimes it is known, often not (blind search).**
- **Frequency:** down, limited by the antenna sensitivity; up to 1~2 kHz; **sometimes it is known , often not (blind search).**
- **Amplitude:**

$$h_0 = 1.05 \cdot 10^{-27} \left(\frac{I_3}{10^{38}\, kg \cdot m^2} \right) \left(\frac{10kpc}{r} \right) \left(\frac{\nu}{100Hz} \right)^2 \left(\frac{\varepsilon}{10^{-6}} \right)$$

I_3 is the principal moment of inertia along the rotation axis, ε is the ellipticity $(I_2-I_1)/I_3$

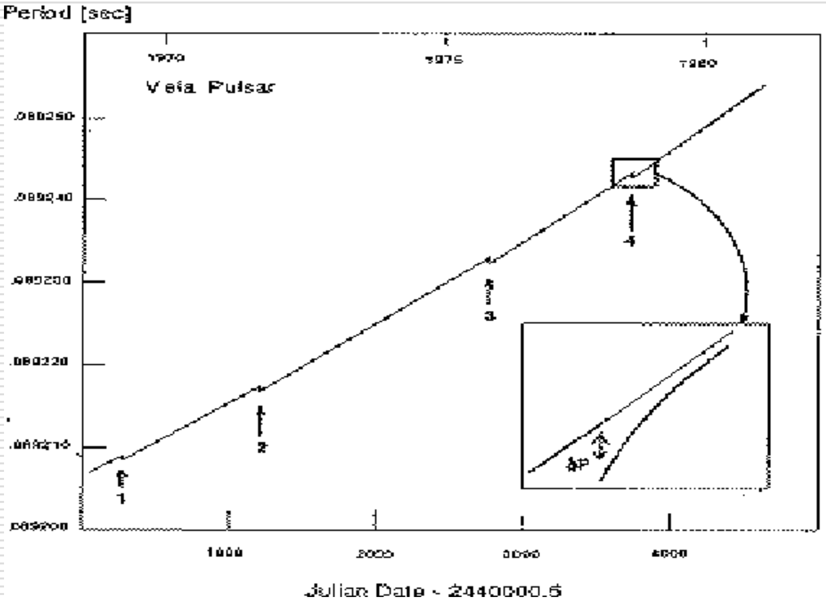
Signal characterization – other features

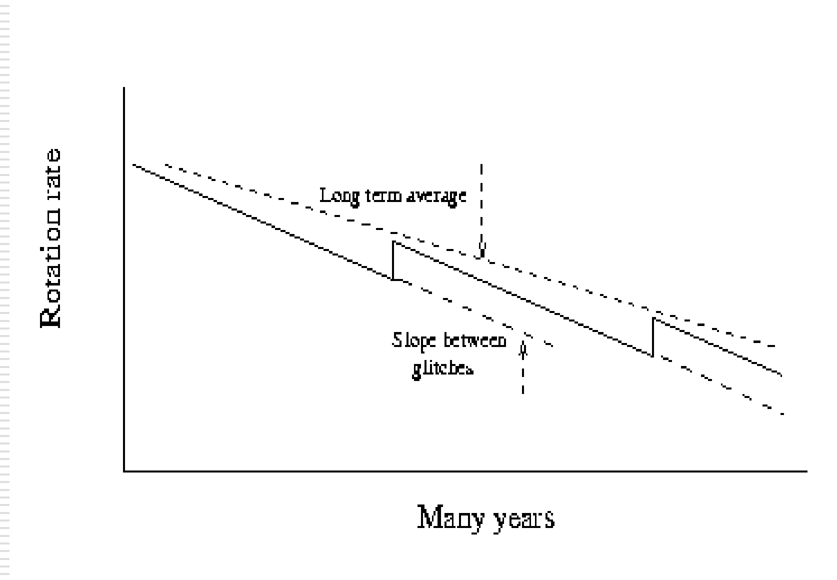
- Doppler frequency modulation, due to the motion of the detector
- Spin-down (or even spin-up), roughly slow exponential
- Intrinsic frequency modulation, due to a companion, an accretion disk or a wobble
- Amplitude modulation, due to the motion of the detector and its radiation pattern and possibly to intrinsic effect (e.g. a wobble)
- Glitches



(X4,Y4,Z4)

Glitches





In the first figure there is the period variation of the Vela Pulsar during about 12 years.

In the second figure there is a more schematic view (frequency on ordinates).

The frequency of glitches depends on the age of the star (younger stars have more glitches) and is not a general feature.

Glitches are related to star-quakes.

Odd features of the data that complicate the detection (in particular of the noise)

- Non-stationarity
- Non-gaussianity
- Non-flatness of the spectrum
- Impulsive and burst noise
- "Holes" in the data

Basic detection techniques

- Matched filter
- Lock-in
- Fourier transform and power spectrum
- Autocorrelation
- Non linear methods

Matched Filter

The optimal detection of a signal of known shape, embedded in white gaussian noise is performed by the ***matched filter***, that can be seen as

$$y(t_{obs}) = \int_0^{t_{obs}} x(t) \cdot f(t) \cdot dt$$

Where

x(t)=k*f(t)+n(t)

are the data and f(t) is the shape, normalized such that

$$\int_0^{t_{obs}} f^2(t) \cdot dt = 1$$

Matched filter for a sinusoid

If the data is composed by the sum of a sinusoid and white gaussian noise

$$x(t) = h_0 \sin(\omega_0 t + \varphi_0) + n(t)$$

the matched filter is

$$y(t_{obs}) = \int_0^{t_{obs}} x(t) \cdot \frac{2 \cdot \sin(\omega_0 t + \varphi_0)}{t_{obs}} \cdot dt$$

and the response to the sinusoidal signal is (the normalization is done to obtain h_0)

$$y_{signal}(t_{obs}) = h_0$$

and the variance of the noise is

$$\sigma_n^2 = \frac{2 \cdot |H_n(\omega_0)|^2}{t_{obs}}$$

The signal-to-noise ratio is

$$SNR_{MF} = \frac{h_0 \cdot \sqrt{t_{obs}}}{\sqrt{2} \cdot H_n(\omega_0)} \approx 2.2 \cdot \left(\frac{h_0}{10^{-26}}\right) \left(\frac{H_n(\omega_0)}{10^{-23} Hz^{-1/2}}\right)^{-1} \left(\frac{t_{obs}}{10^7 s}\right)^{\frac{1}{2}}$$

Detection statistics and lock-in

The matched filter is a linear filter, so the noise at the output is gaussian.

If the phase is unknown, the detection can be achieved by a lock-in amplifier (or an equivalent computer algorithm)

$$\xi(t_{obs}) = \int_0^{t_{obs}} x(t') \cdot e^{j\omega_0 t'} \cdot dt'$$

where ω_0 is the tuning angular frequency. In such case the noise power is doubled and its distribution is no more gaussian.

If ω_0 changes in time with a known law, the method works well if we substitute $\omega_0 t'$ with the changing phase $\phi(t')$.

Note that a typical laboratory lock-in has an exponential memory

$$\xi(t) = \int_0^{t} x(t') \cdot e^{j\omega_0 t'} \cdot e^{\frac{t'-t}{\tau}} dt'$$

Power spectrum by FFT periodogram

If the frequency (and the phase) of the signal is not known, the better way to detect a periodic signal is by the **estimate of the power spectrum**. This can be obtained by a **periodogram**, i.e. the square modulus of the Fourier transform of the data.

Remember that the power spectrum is, by definition, the Fourier transform of the **autocorrelation** $R_{xx}(\tau)=E[x(t)x(t+\tau)]$.

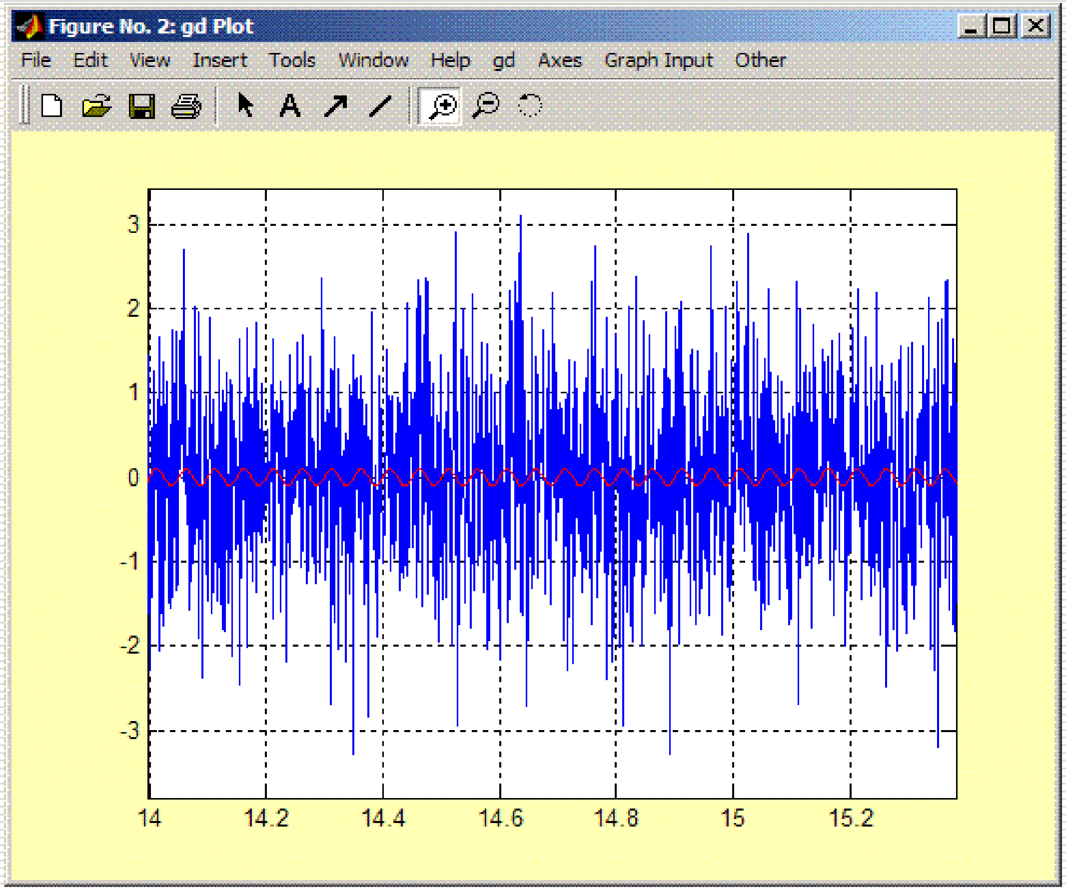
PS with FFT periodogram is like a set of lock-in at n frequencies.

An efficient algorithm to compute the (discrete) Fourier transform of the sampled data is the *Fast Fourier Transform* (FFT). The number of floating point operations (FLOP) needed to compute an FFT of length n (that should be a power of 2) is about

$$5 * n * \log_2(n)$$

instead of something proportional to n^2.

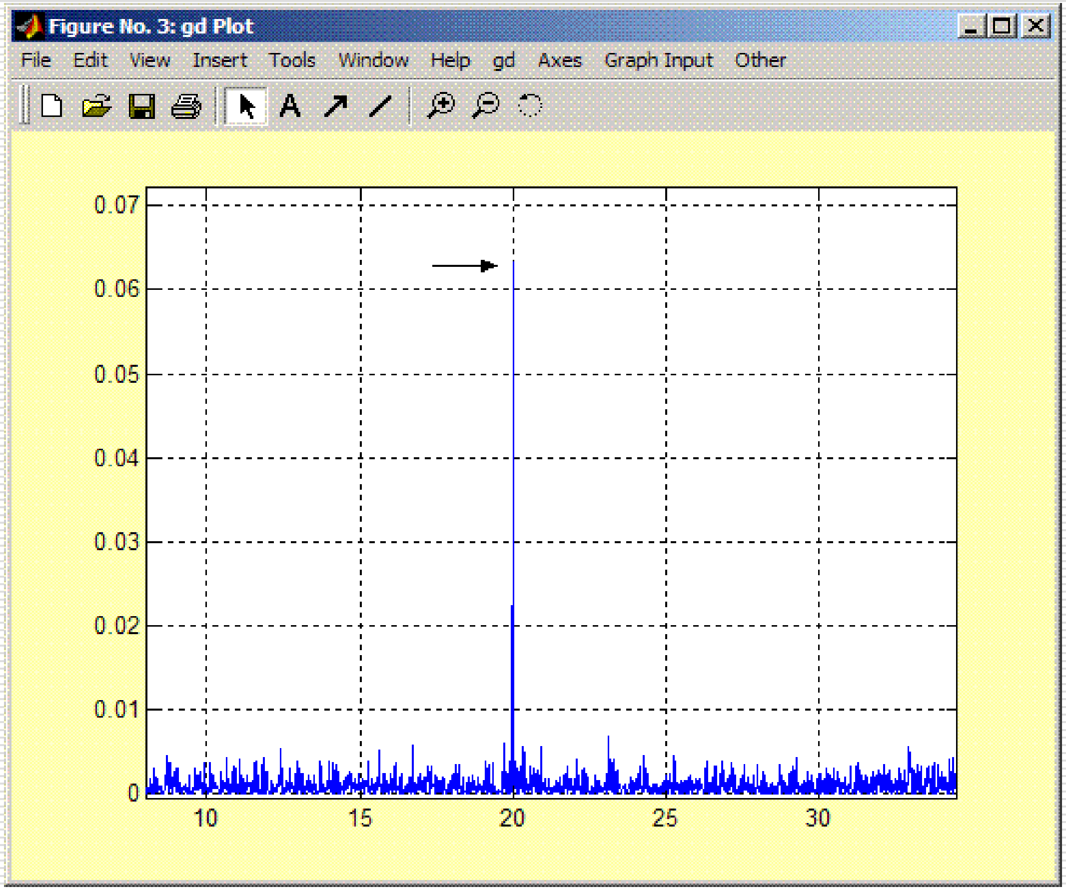
Signal and noise



The noise is white gaussian with standard deviation equal to 1

The signal (in red) is a sinusoid of amplitude 0.1 and frequency of 20 Hz

Power Spectrum Estimation



This is the power spectrum of the previous signal+noise, estimated by the FFT periodogram (length 32768 = 2^{15}).

The arrow indicates the signal peak at 20 Hz

How this detection happens ? The point is that the noise power is spread in all the spectrum bins, while the signal goes only in one.

Power spectrum by FFT periodogram (some details)

Discrete Fourier transform

$$X_k = \sum_{i=0}^{n-1} x_i \cdot e^{j \cdot \frac{k \cdot i}{n}}$$

Frequency resolution

(if the signal power goes all in a single bin, the noise power in the bin is proportional to the bin width)

$$\Delta \nu = \frac{1}{T_{obs}}$$

Signal-to-noise ratio (linear)

$$SNR_{PS} = \frac{h_0 \cdot \sqrt{t_{obs}}}{2 \cdot H_n(\omega_0)} \approx 1.6 \cdot \left(\frac{h_0}{10^{-26}} \right) \left(\frac{H_n(\omega_0)}{10^{-23} Hz^{-1/2}} \right)^{-1} \left(\frac{t_{obs}}{10^7 s} \right)^{\frac{1}{2}}$$

$\sqrt{2}$ less than the SNR of the matched filter

Power spectrum as the mean of periodograms

The distribution of the amplitude of the bins of the periodogram of a chunk of white gaussian noise is exponential. It remains exactly the same increasing the length of the periodogram, and the same is obviously for the mean and the variance.

To reduce the variance of the noise spectrum, one way is by dividing the chunk of data in **N** pieces, take the periodograms of each piece and then make the average.

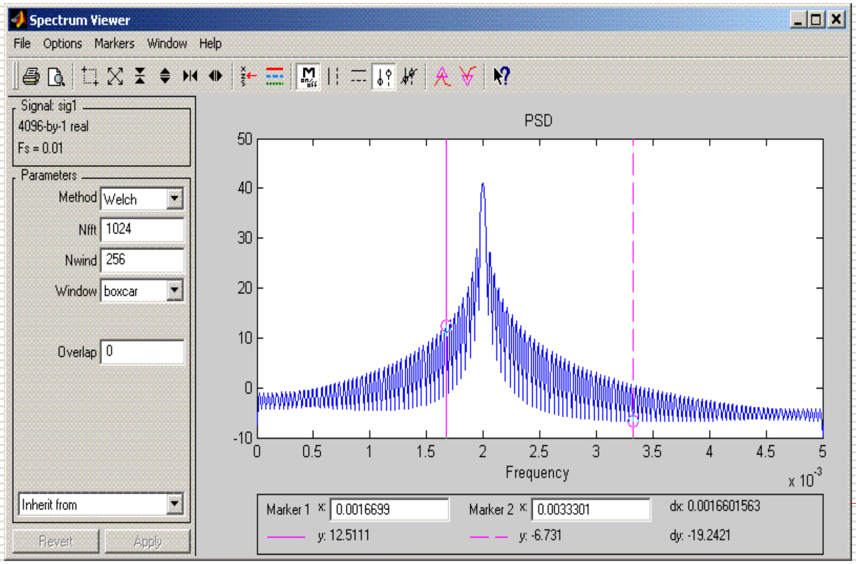
In this case both the variance and the signal is reduced and the (linear) SNR is reduced by a factor $\sqrt[4]{N}$.

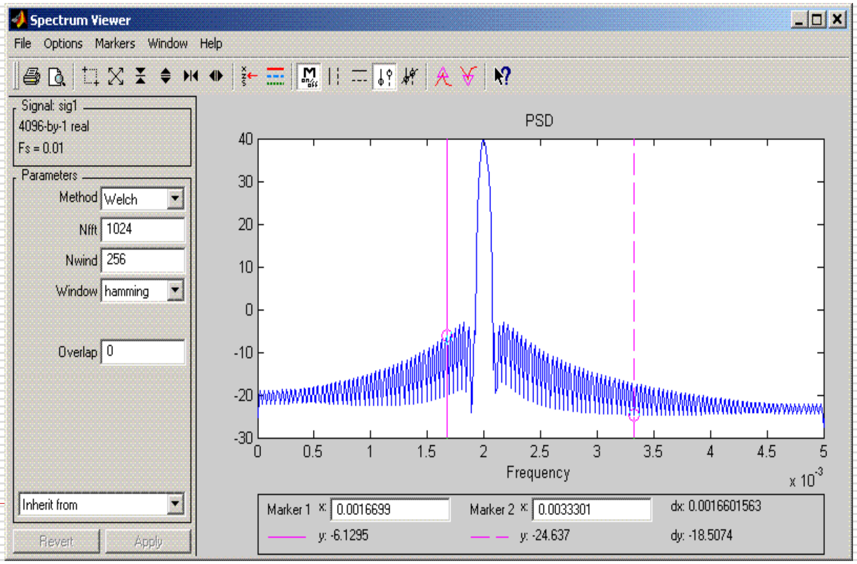
The distribution is a χ^2 with 2N degrees of freedom.

Windowing

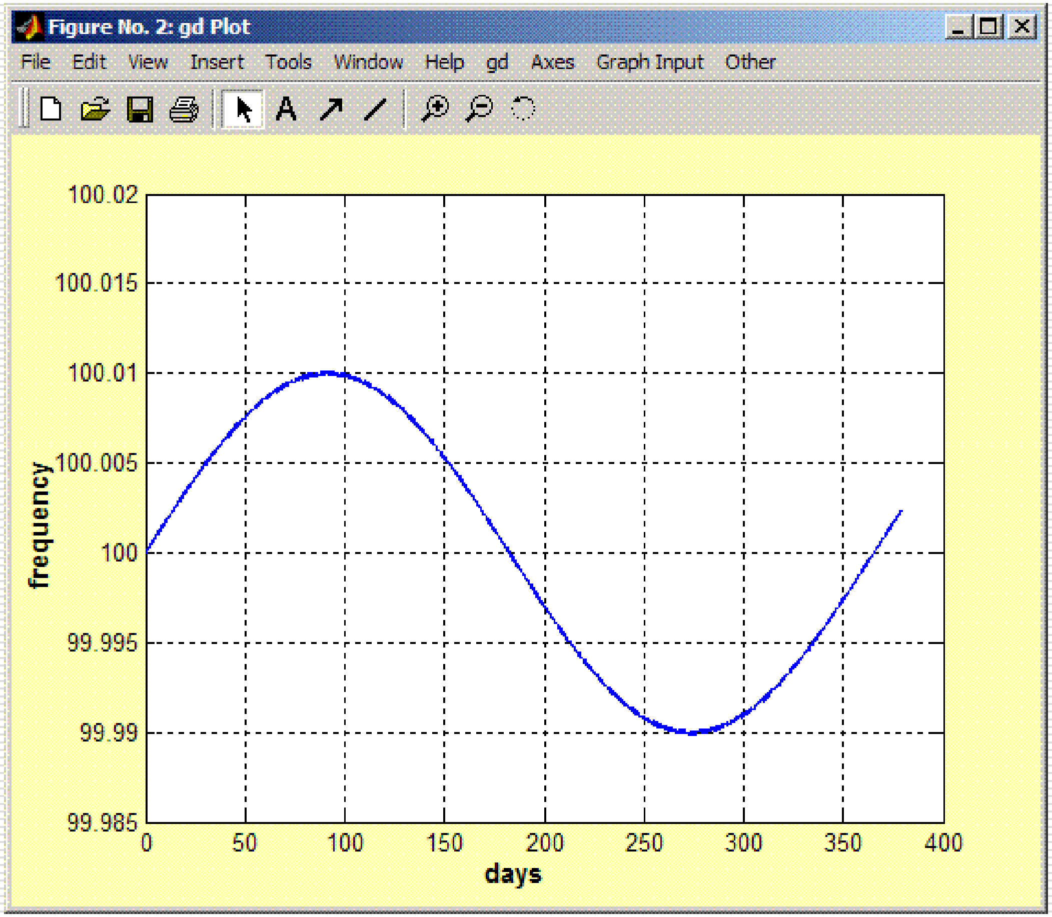
When the frequency of a sinusoidal signal has not exactly the central value of a frequency bin of a periodogram, the energy of the signal goes also in other bins. To reduce this effect, special weighting functions, called windows, have been studied. The use of a window normally reduces the resolution.

In the two figures we see the effect of two different windows on the power spectrum estimate of the same signal.





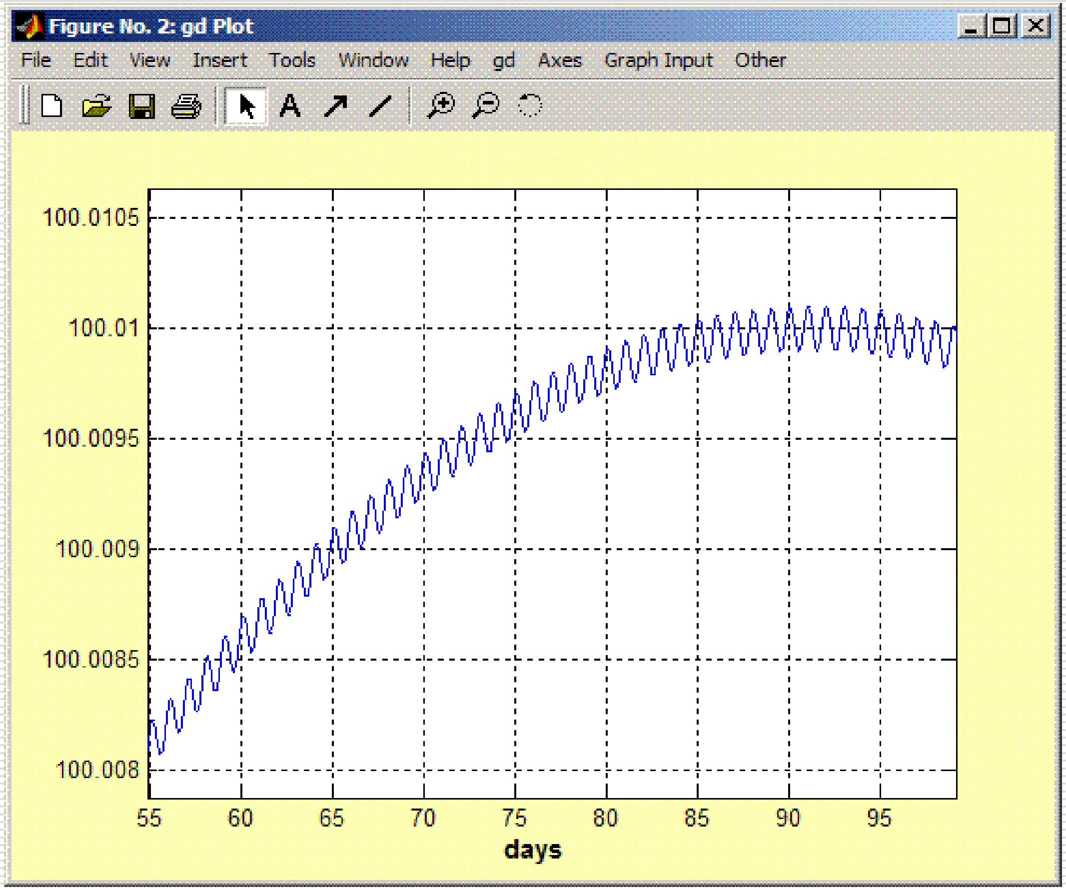
Doppler effect



The Doppler variation of the frequency in a period of one year for a low (ecliptical) latitude source.

The original frequency is 100 Hz and the maximum variation fraction is of the order of 0.0001

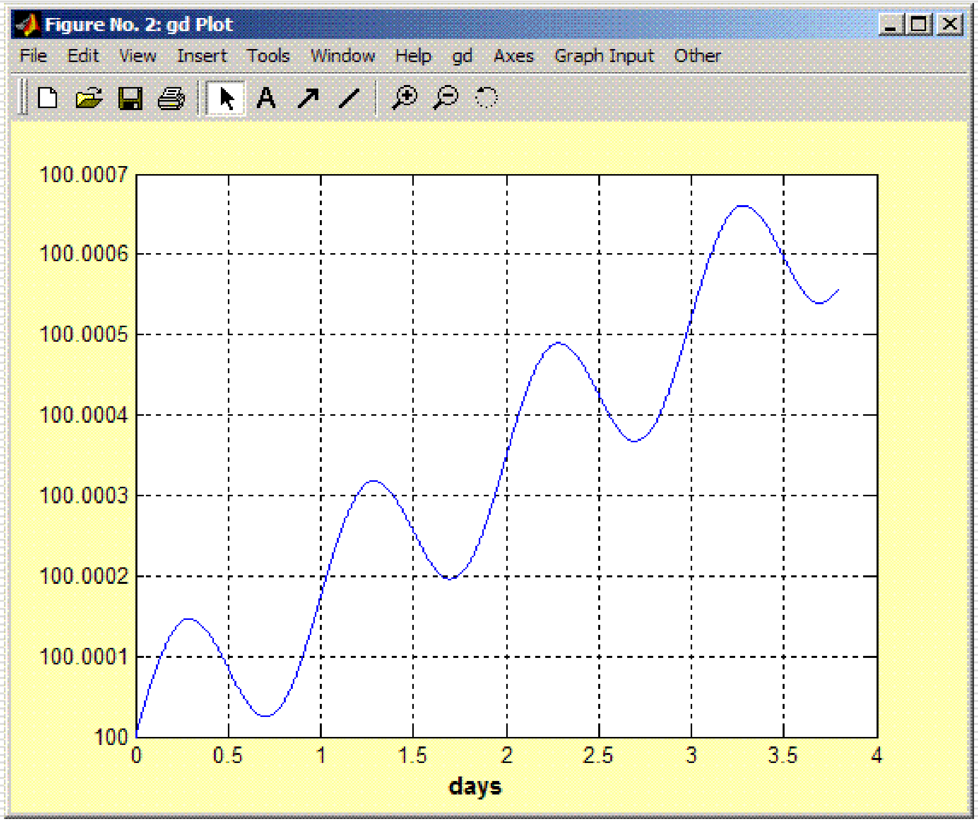
Doppler effect - zoom



Zoom of the preceding figure. Note the daily variations.

Very roughly the Doppler effect can be seen as the sum of two “epicycles” (Ptolemaic view)

Doppler effect (zoom)



Frequency variation on about 4 days.

Note that the problem is not in the presence of the Doppler shift, but in the time variation of the Doppler shift. So the effect of the rotation is more relevant of that of the orbital motion. The rotation epicycle is dominant.

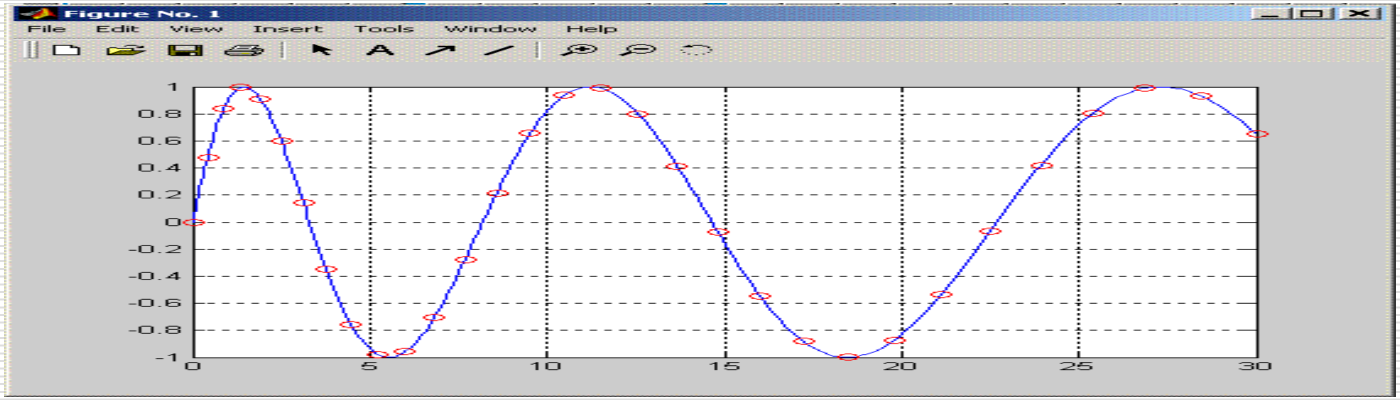
Optimal detection by re-sampling procedure

Because of the frequency variation, the energy of the wave doesn’t go in a single bin, so the SNR is highly reduced.

A solution to the problem of the varying frequency is to use a non-uniform sampling of the received data: if the sampling frequency is proportional to the (varying) received frequency, the samples, seen as uniform, represent a constant frequency sinusoid and the energy goes only in one bin of their FFT.

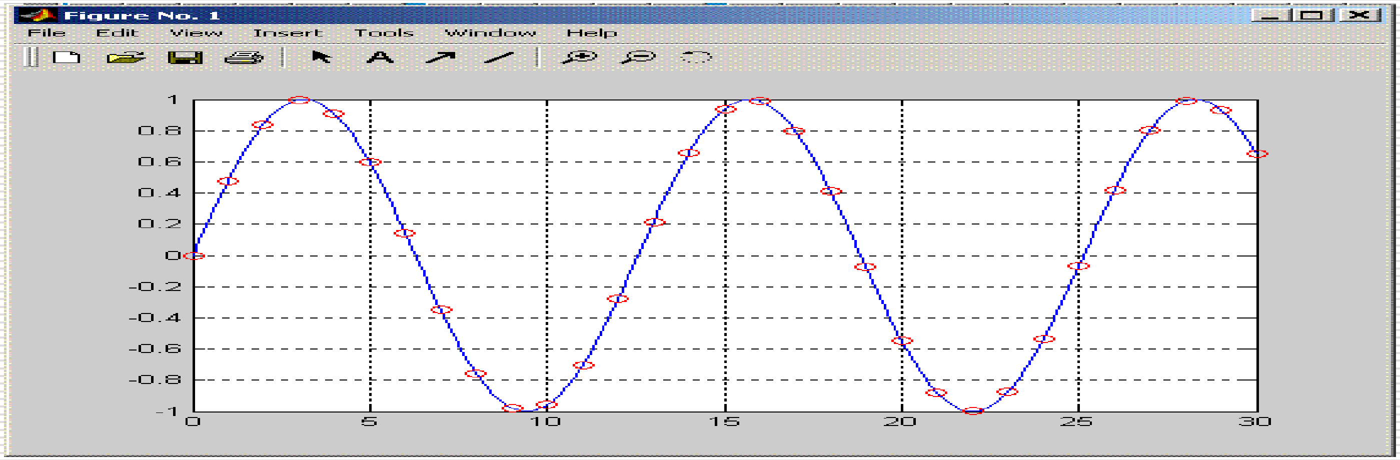
Every point of the sky (and every spin-down or spin-up behavior) needs a particular re-sampling and FFT.

Resampling



Original data:

The frequency is varying, we sample non-uniformly (about 13 samples per period).



The non-uniform samples, seen as uniform, give a perfect sinusoid and the periodogram of the samples has a single “excited” bin.

Optimal detection

	1 month	4 months	1 year
FFT length (number of points)	2.6E+09	1.0E+10	3.1E+10
Sky points	2.1E+11	3.4E+12	3.1E+13
Spin-down points (1st)	2.2E+04	3.5E+05	3.1E+06
Spin-down points (2nd)	1.0E+00	1.1E+01	3.2E+02
Freq. points (500 Hz)	1.3E+09	5.0E+09	1.6E+10
Total points	6.0E+24	6.5E+28	4.8E+32
Comp. power (Tflops)	1.0E+12	1.5E+16	3.6E+19
Sensitivity (nominal)	1.2E-26	6.0E-27	3.6E-27
(background 10^-23*Hz^-0.5)			
Sens. for 10^9 candidates	7.4E-26	4.1E-26	2.7E-26

It is supposed a 2 kHz sampling frequency. For the computation power, an highly optimistic estimation is done and it is not considered the computation power needed by the re-sampling procedure. The decay time (spindown) is taken higher than 10^4 years.

Some concepts and numbers on computing power

- The "crude" computing power of a computer system is often expressed in FLOPS (floating point operations per second)
- A today (2004) workstation has a computing power of 3 GFLOPS (10^9 FLOPS)
- A today big supercomputer (a cluster of many PCs or a server with many CPUs) has a computing power of about 10 TFLOPS (10^{13} FLOPS)
- The Moore Law says that the computing power of standard computers doubles every 1.5 years
- The crude computing power may be not meaningful, because in many algorithms (the vast majority) the access time to the RAM or to the disk is dominant.
- The problem is not only to have a big computer, but also have an algorithm that exploits at best its architecture and minimizes the accesses to RAM and disk.

Introduction to the hierarchical search

- Because the "optimal detection" cannot be done in practice, we have proposed the use of a sub-optimal method, based on alternating "incoherent" and "coherent" steps
- The first incoherent step consist of Hough or Radon transform based on the collection of short FFT periodograms. From this step we "produce" candidates of possible sources
- Then, with a coherent step, we "zoom" on the candidates, refining the search
- Then a new incoherent step can be done, and so on, until the full sensitivity is reached

Short periodograms and short FFT data base

- The basis of the hierarchical search method is the "short FFT data base"
- It is used for producing the periodograms for the incoherent steps and the data for the coherent step
- How long should be a "short FFT" ?

Short periodograms and short FFT data base (continued)

What is the maximum time length of an FFT such that a Doppler shifted sinusoidal signal remains in a single bin ? (Note that the variation of the frequency increases with this time and the bin width decreases with it)

The answer is

$$T_{\max} = T_E \cdot \sqrt{\frac{c}{4\pi^2 R_E \nu_G}} \approx \frac{1.1 \cdot 10^5}{\sqrt{\nu_G}}\ s$$

where T_E and R_E are the period and the radius of the "rotation epicycle" and ν_G is the maximum frequency of interest of the FFT.

Short periodograms and short FFT data base (continued)

As we will see, we will implement an algorithm that starts from a collection of short FFTs (the SFDB, short FFT data base).

Because we want to explore a large frequency band (from ~10 Hz up to ~2000 Hz), the choice of a single FFT time duration is not good because, as we saw,

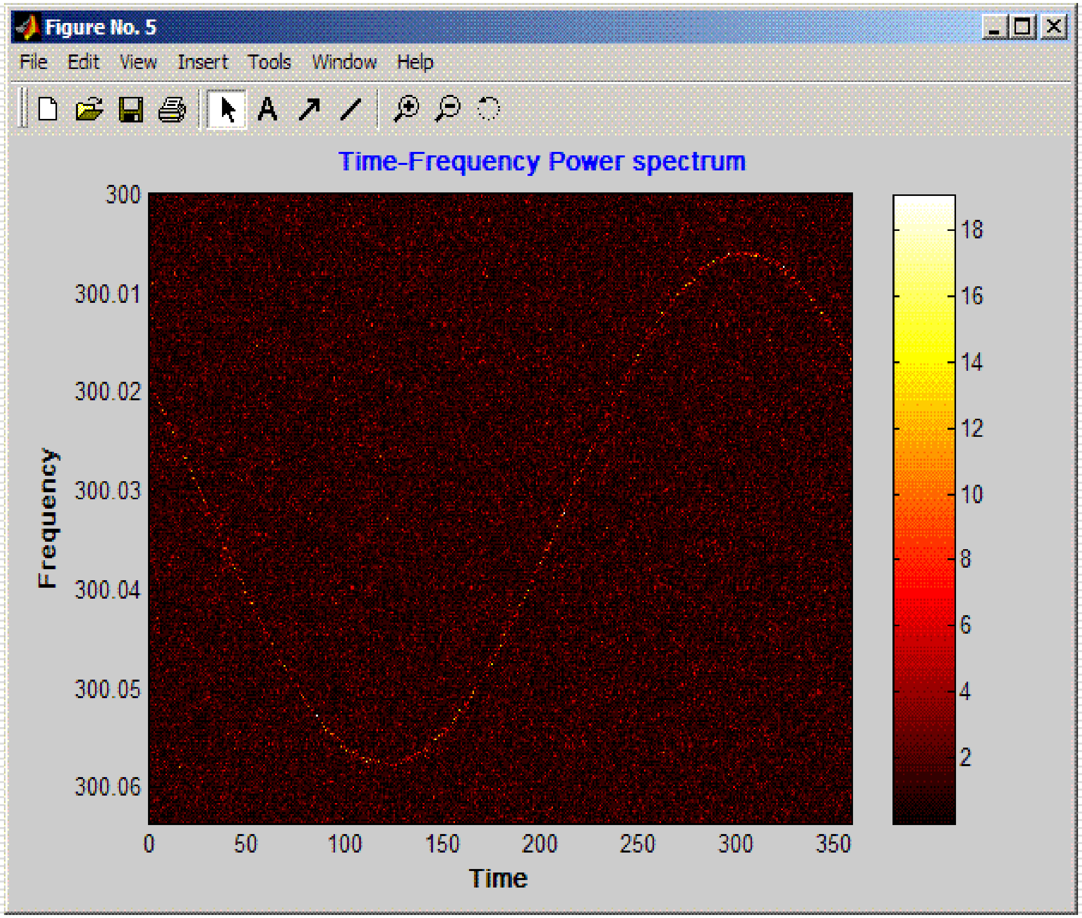
$$T_{\max} \propto \nu_G^{-\frac{1}{2}}$$

so we propose to use 4 different SFDB bands.

The 4 SFDB bands

	Band 1	Band 2	Band 3	Band 4
Max frequency of the band (Nyquist frequency)	2000	500	125	31.25
Observed frequency bands	1500	375	93.75	23.438
Max duration for an FFT (s)	2445	4891	9782	19565
Length of the FFTs	4194304	4194304	2097152	1048576
FFT duration (s)	1048	4194	8388	16777
Number of FFTs (4 months)	20063	5015	2508	1254
SFDB storage (GB; one year)	510	130	33	9

Radon transform (stack-slide search)



Here is a time-frequency power spectrum, composed of many periodograms (e.g. of about one hour).

In a single periodogram the signal is low, and so is for the average of all the periodogram, but if one shift the periodograms in order to correct the Doppler effect and the spin-down, and then take the average, we have a single big peak.

In this case, for the average of n periodograms, the noise has a chi-square distribution with 2*n degrees of freedom (apart for a normalization factor)

Radon transform (reference)

Johann Radon, "Uber die bestimmung von funktionen durch ihre integralwerte langs gewisser mannigfaltigkeiten (on the determination of functions from their integrals along certain manifolds," Berichte Saechsische Akademie der Wissenschaften, vol. 29, pp. 262 - 277, 1917.

Johann Radon was born on 16 Dec 1887 in Tetschen, Bohemia (now Decin, Czech Republic) and died on 25 May 1956 in Vienna, Austria

Hough transform

Another way to deal with the changing frequency signal, starting from a collection of short length periodograms, is the use of the Hough transform (see **P.V.C. Hough, "Methods and means for recognizing complex patterns", U.S. Patent 3 069 654, Dec 1962**)

Linear Hough transform

Suppose to have an image of one particle track in a bubble chamber, i.e. a number of aligned points together with some random points. The problem is to find the parameters **p** and **q** of a straight line

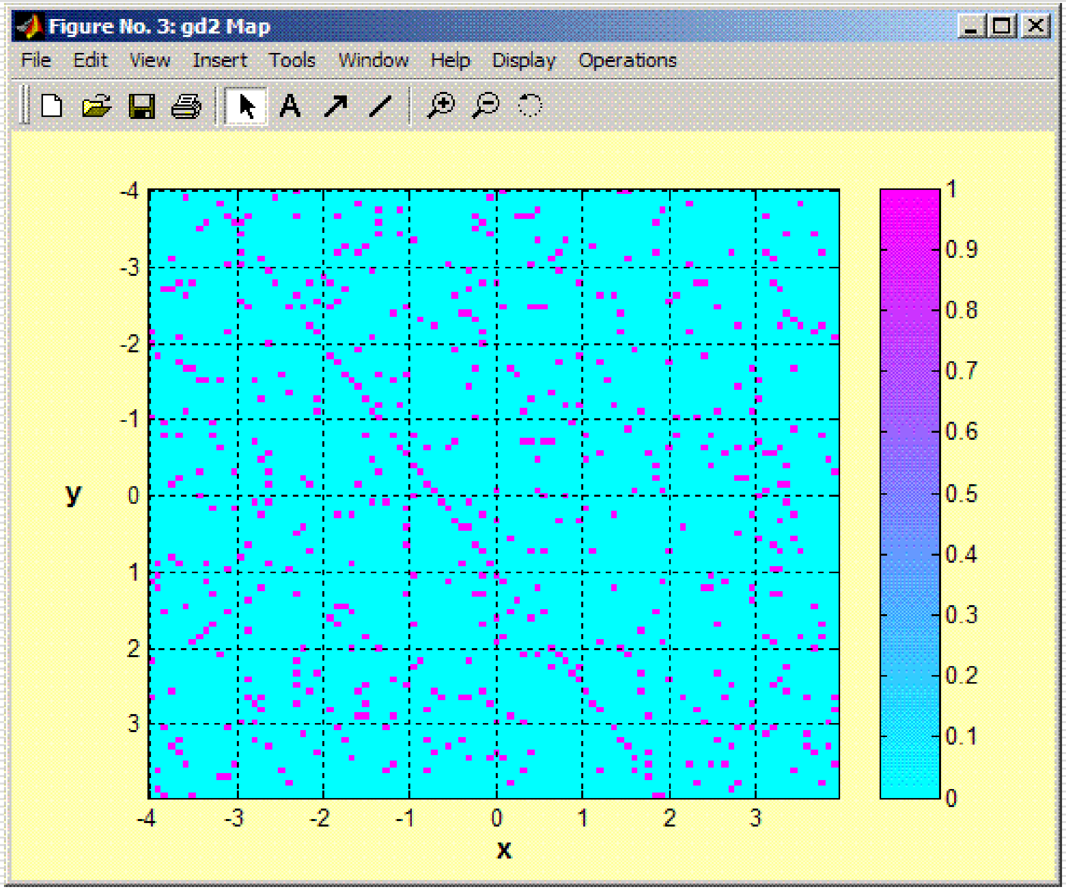
$$y = p * x + q$$

The "Hough transform" transform each point in the plane **(x,y)** to a straight line

$$q = - x * p + y$$

in the plane **(p,q)** and conversely a straight line in the **(x,y)** plane to a point in the **(p,q)** plane: the coordinate of the point in this plane are the parameters of the straight line.

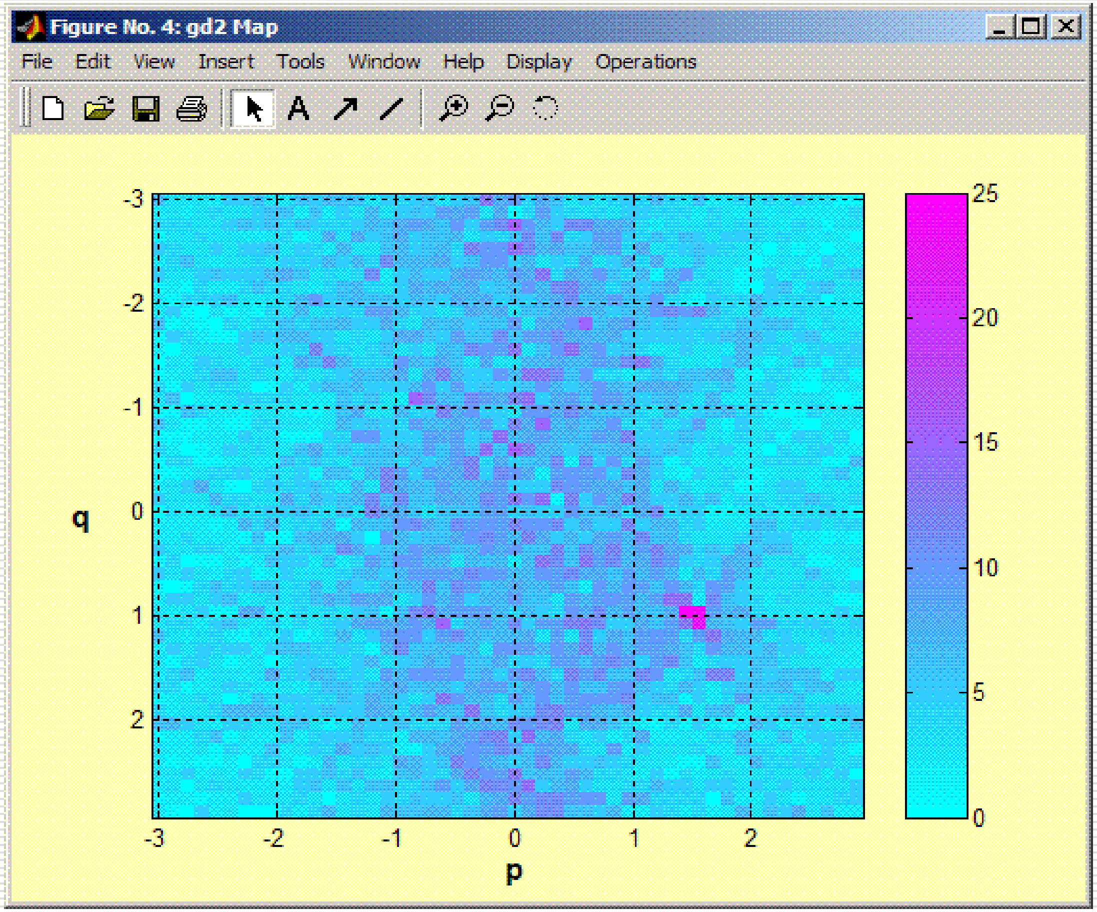
Peak Map - 1



Peak map (or bubble chamber image) with a straight line with equation

$$y = 1.5 * x + 1$$

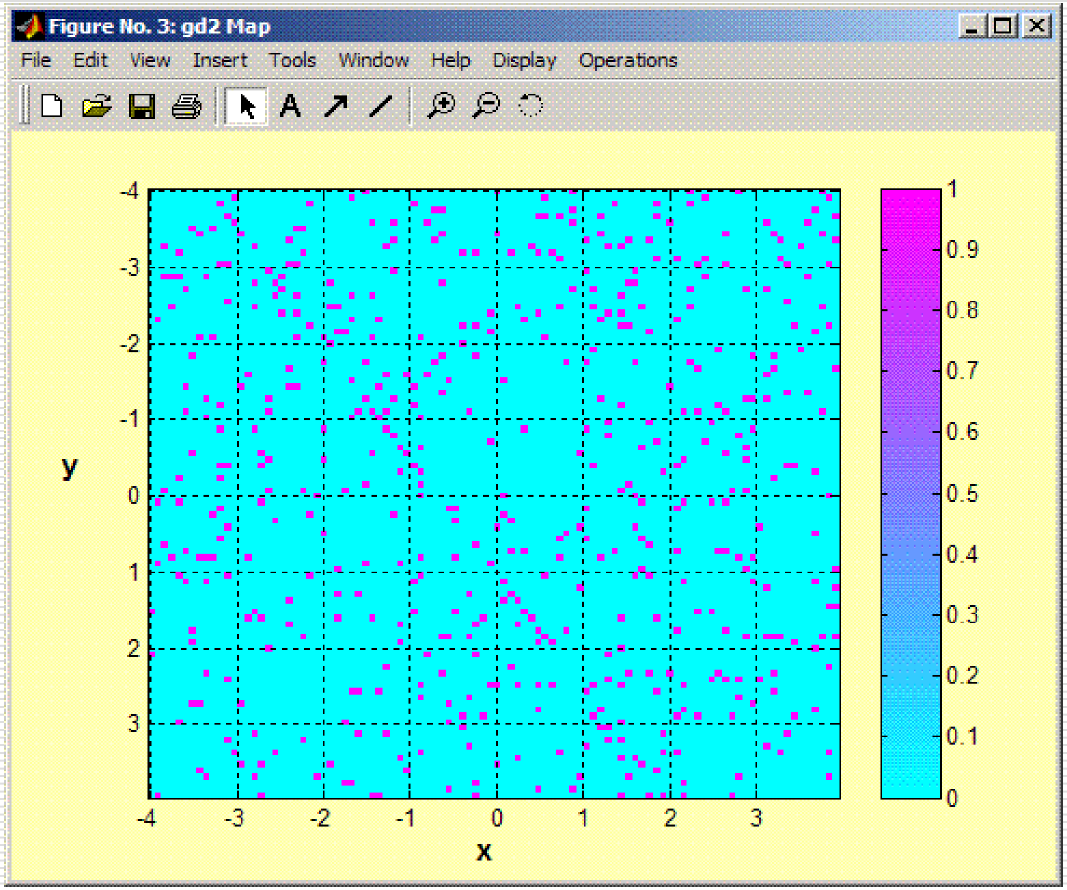
Hough Map - 1



Hough map of the preceding image. This can be seen as a 2-dimension histogram: for each point in the peak map, a set of aligned bins representing a straight line in the Hough map is increased by 1.

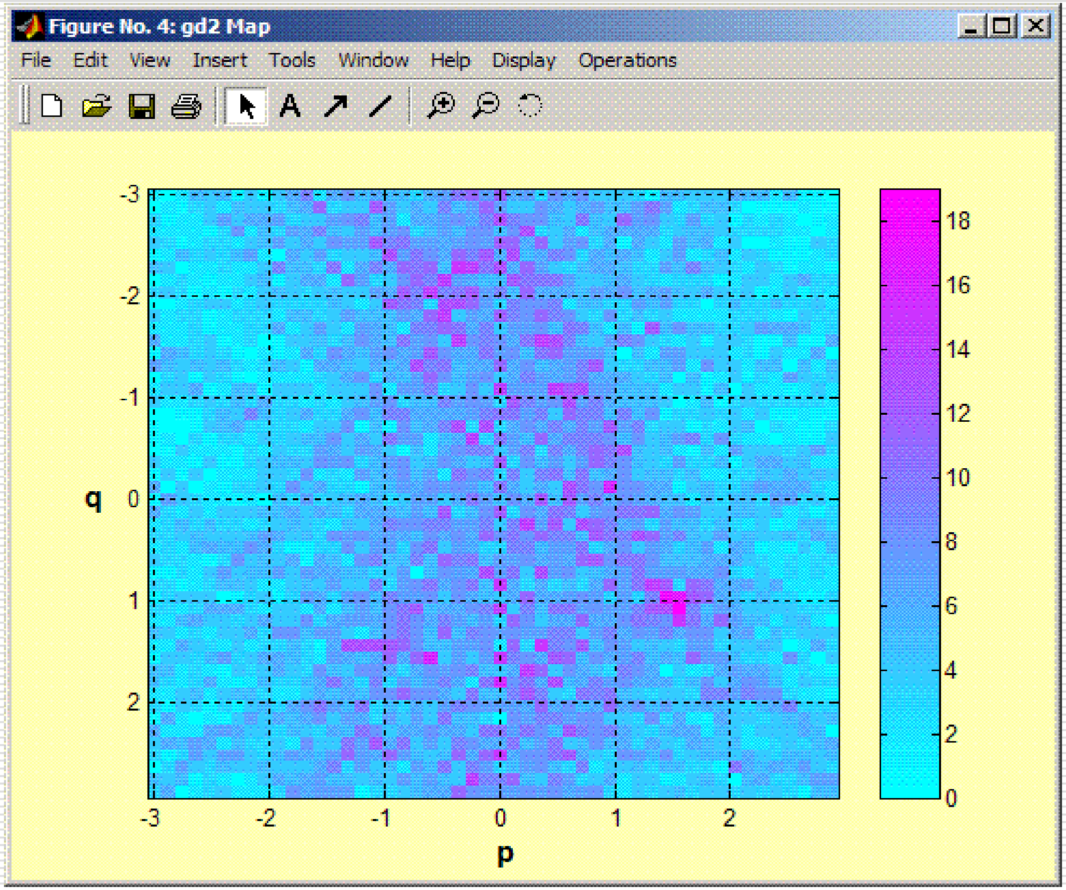
Note the peak at about p = 1.5 and q = 1

Peak Map - 2



The same of the preceding peak map, but with lower SNR (signal-to-noise ratio)

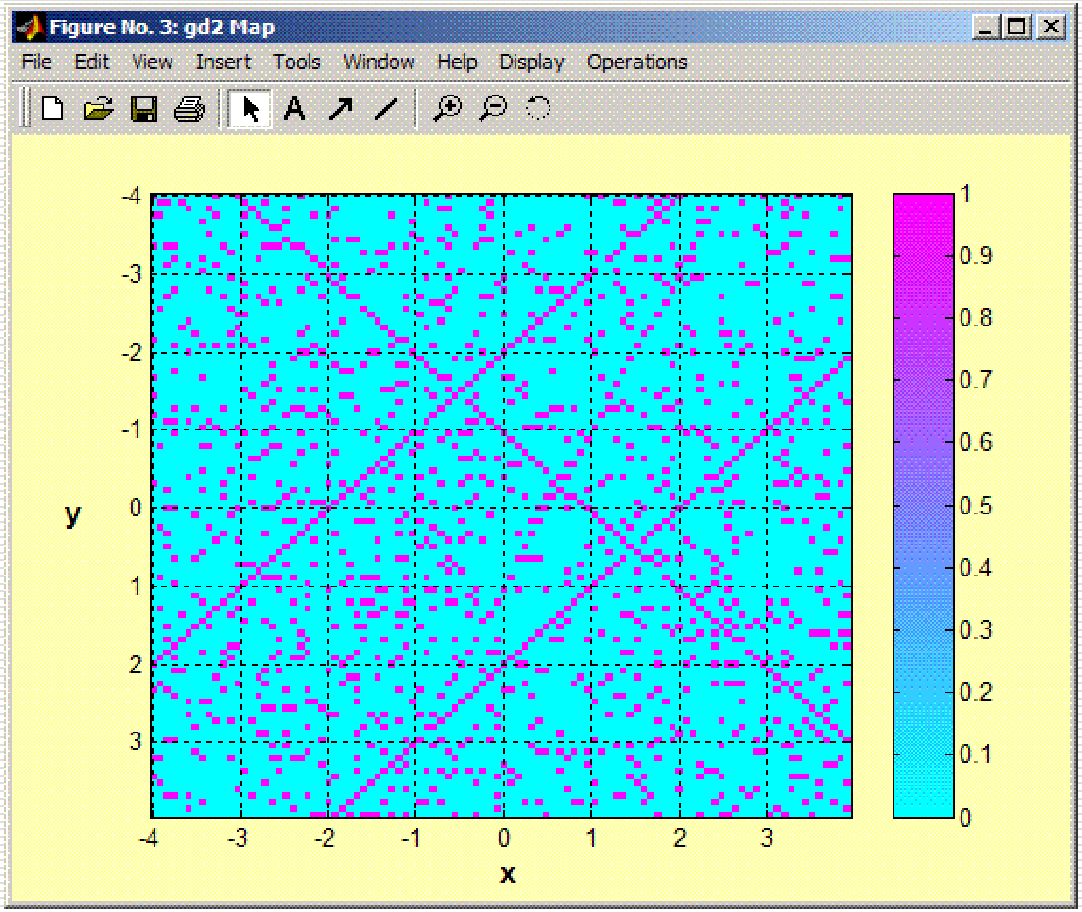
Hough map - 2



More noisy Hough map. The peak is always present, but there are also others, spurious.

Note that the noise is not uniform on the whole map.

Peak Map - 3



Peak map with 4 straight lines:

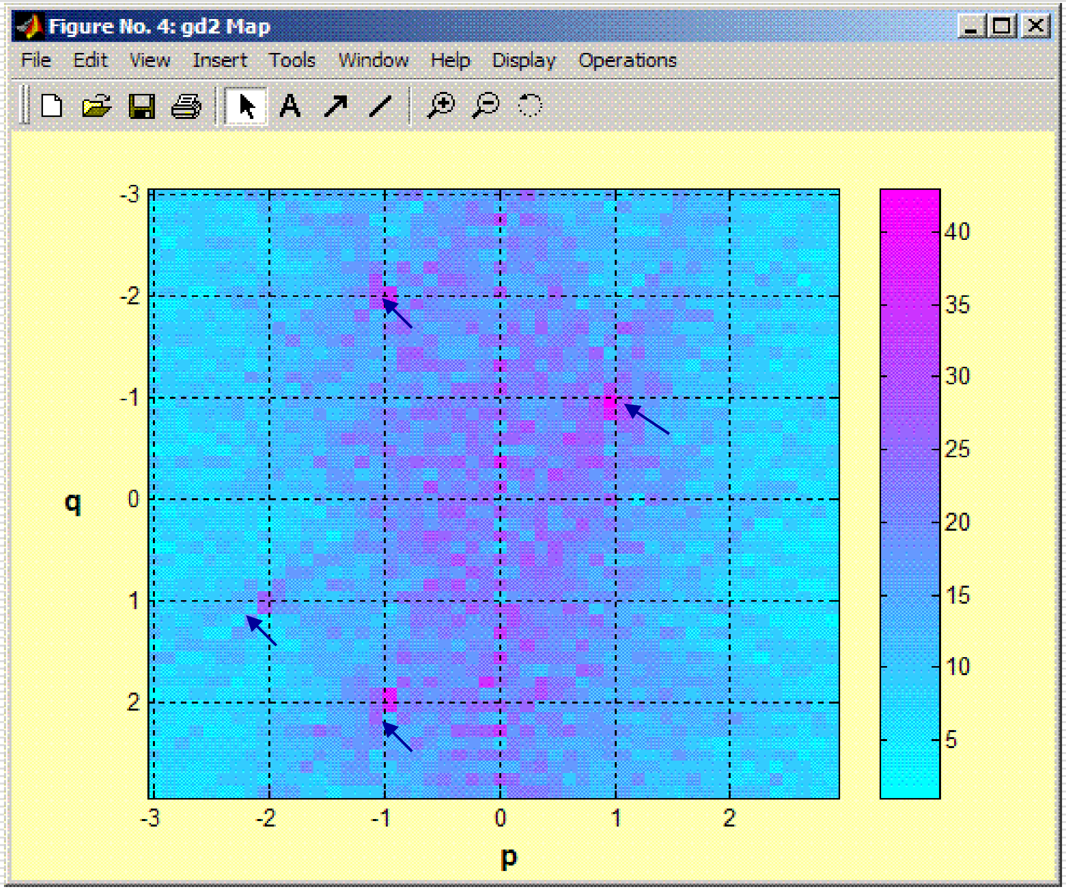
$$y = -x + 2$$

$$y = -x - 2$$

$$y = x - 1$$

$$y = -2 * x + 1$$

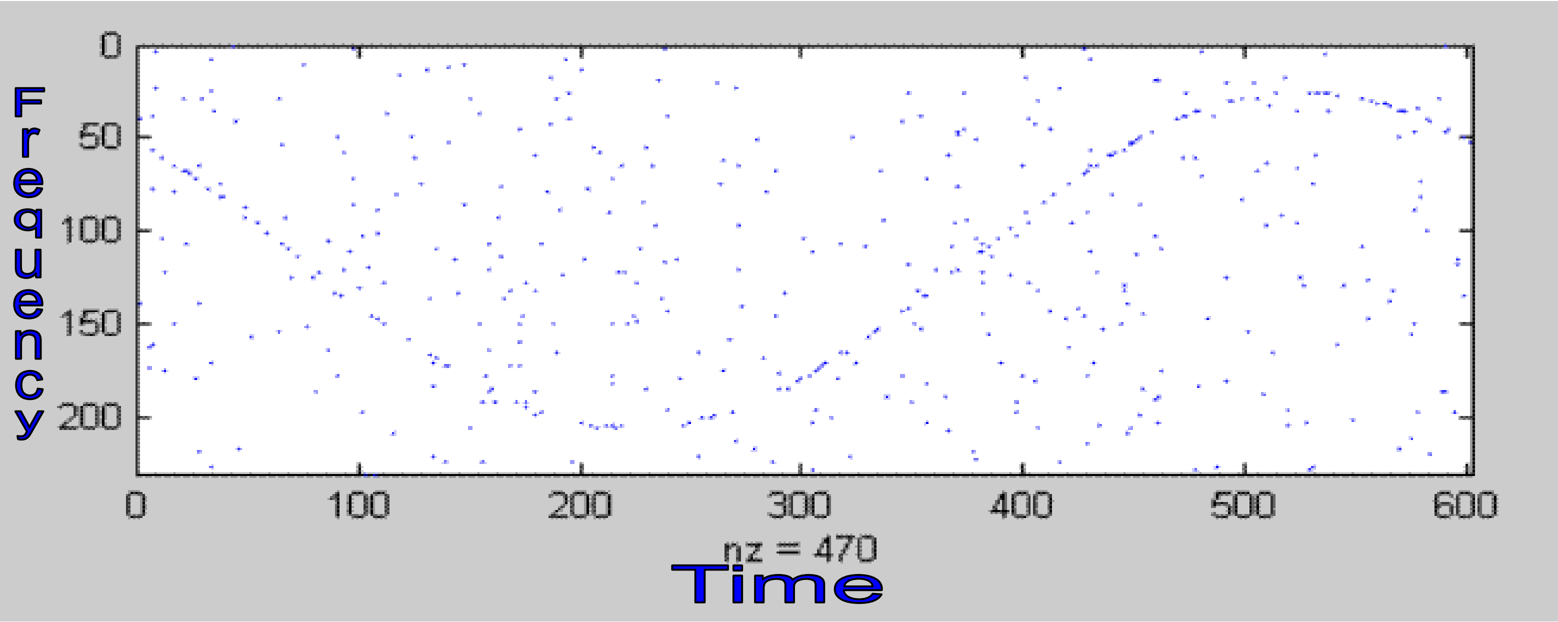
Hough map - 3



All the 4 straight line have been detected, with correct parameters.

Time-frequency peak map

Using the SFDB, create the periodograms and then a time-frequency map of the peaks above a threshold (about one year observation time).
Note the Doppler shift pattern and the spurious peaks.



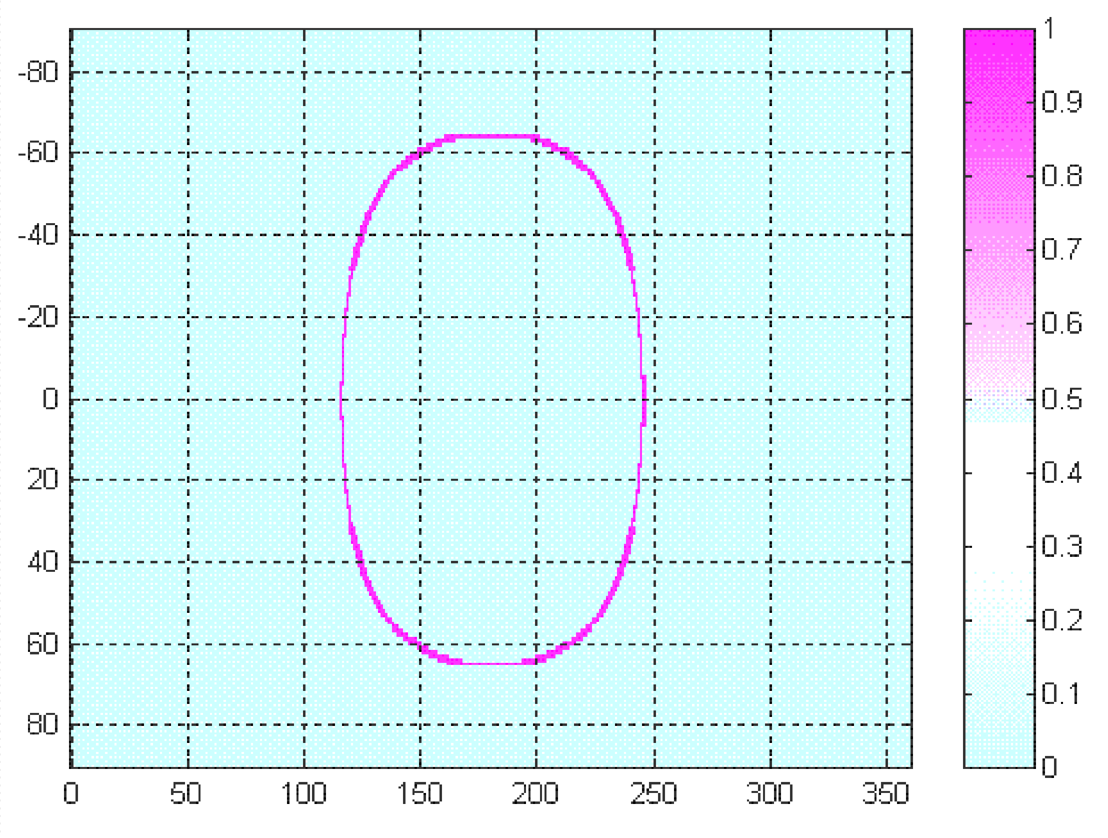
Celestial coordinates Hough map

The Hough transform answers the question:

- Which is the place of the sky from where the signal comes, given a certain Doppler shift pattern ?

It maps the peaks of the time-frequency power spectrum (peak map) to the set of points of the sky.

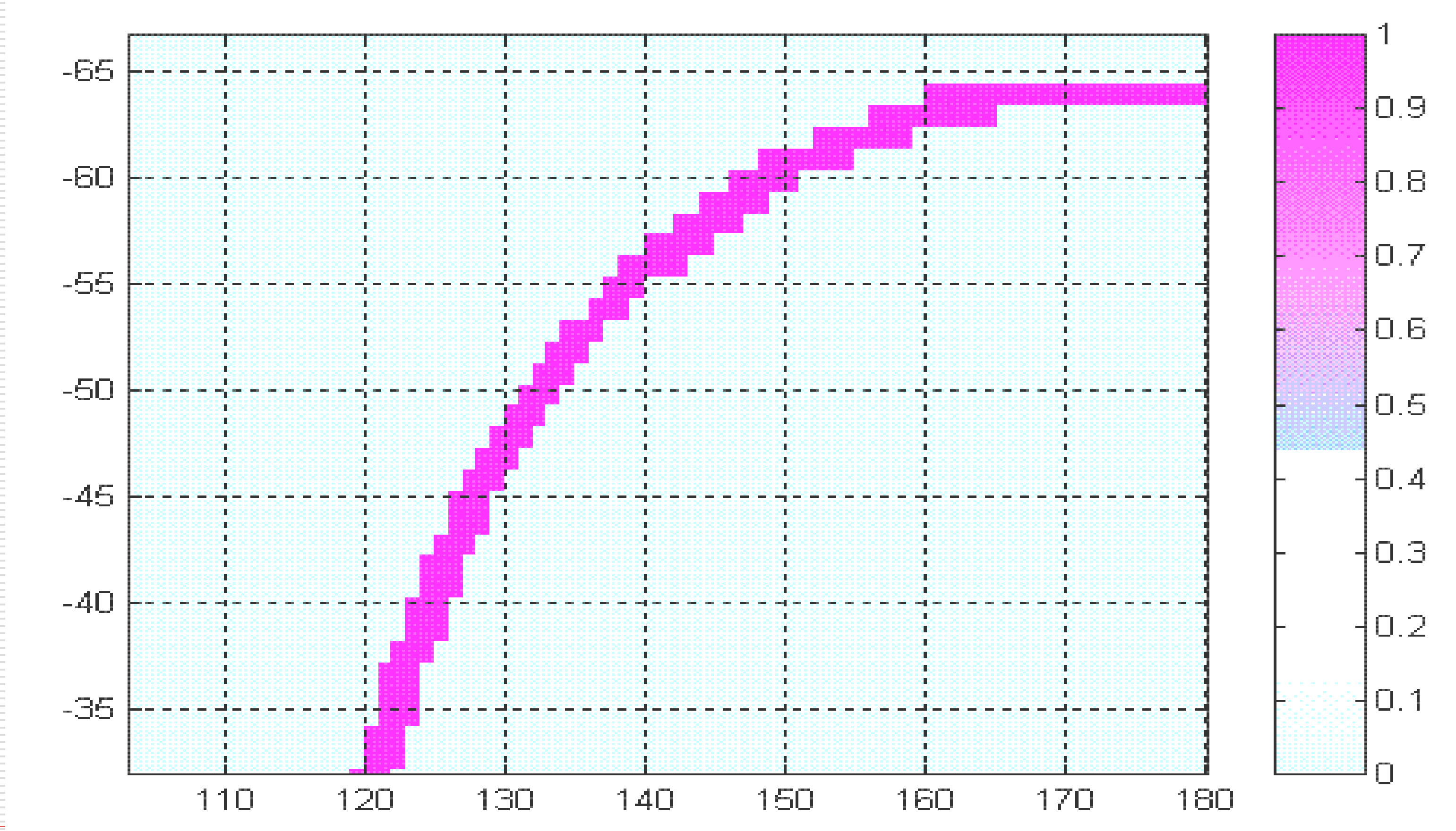
Hough map – single annulus



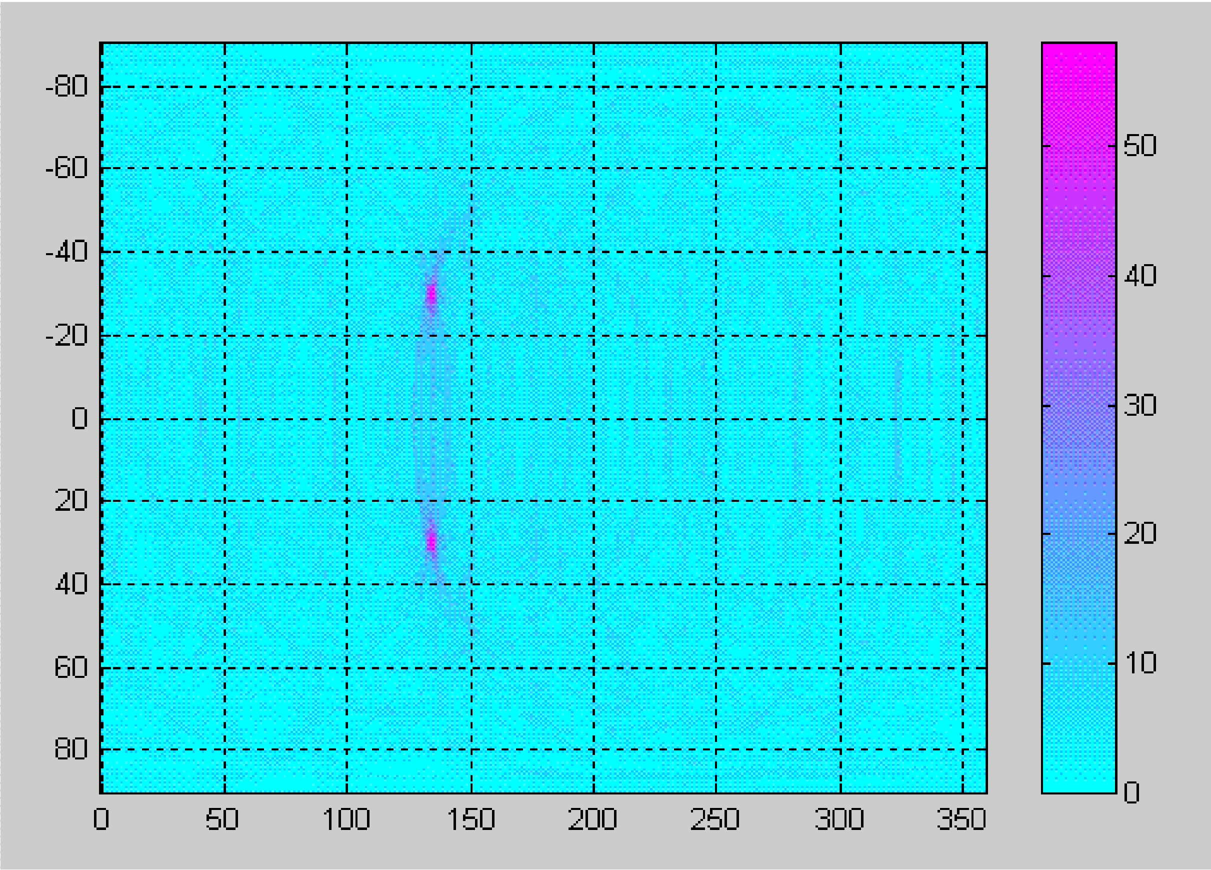
Suppose you are investigating on the possibility to have a periodic wave at a certain frequency.

For every peak in the time-frequency map (in the range of the possible Doppler shift), we take the locus of the points in the sky that produce the Doppler shift equal to the difference between the supposed frequency and the frequency of the peak. Because of the width of the frequency bins, this is not a circle in the sky, but an annulus.

Hough map – single annulus (detail)



Hough map – source reconstruction

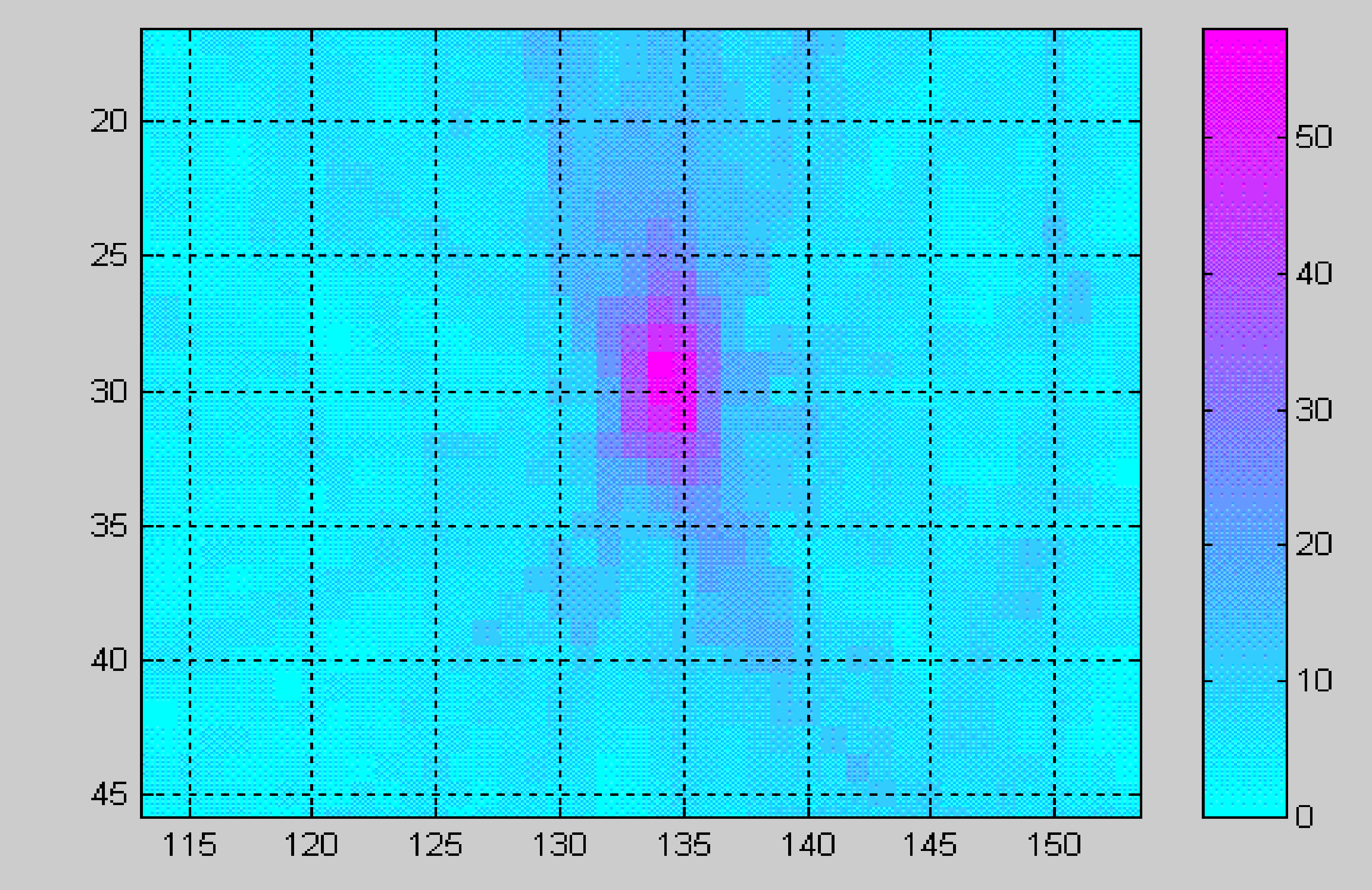


For every peak, we compute the annulus and enhance by one the relative pixels of the sky map.

Doing the same for all the peaks, we have a two-dimension histogram, with one big peak at the position of the source.

Normally, because the motion of the detector that has a big component on the ecliptical plane, there is also a “shadow” false peak, symmetrical respect this plane.

Source reconstruction - detail



Time-frequency power spectrum Hough transform (summary)

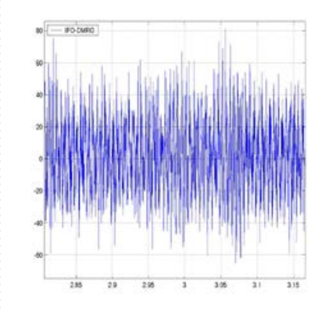
- using the SFDB, create the periodograms and then a time-frequency map of the peaks above a given threshold

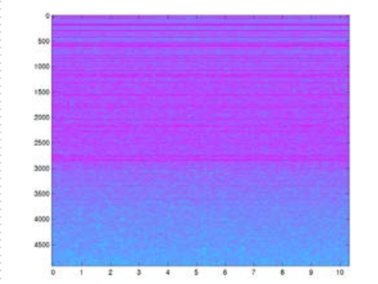
- for each spin-down parameter point and each frequency value, create a sky map ("Hough map"); to create a Hough map, sum an annulus of "1" for each peak; an histogram is then created, that must have a prominent peak at the "source"

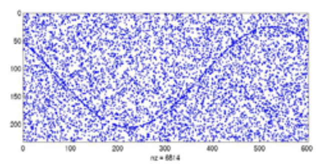
Hierarchical method

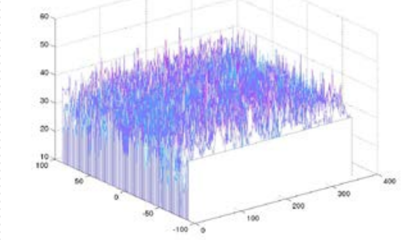
- Divide the data in (interlaced) chunks; the length is such that the signal remains inside one frequency bin
- Do the FFT of the chunks; this is the SFDB
- Do the first "incoherent step" (Hough or Radon transform) and take candidates to follow
- Do the first "coherent step", following up candidates with longer "corrected" FFTs, obtaining a refined SFDB (on the fly)
- Repeat the preceding two step, until we arrive at the full resolution

Hierarchical search









Data -> SFDB -> peak map -> Hough map

Then :

-select candidates on Hough map (with a threshold)

-zoom on data with the “known” parameters

-repeat the procedure with zoomed data, increasing the length of the FFT in steps, until the maximum sensitivity is reached

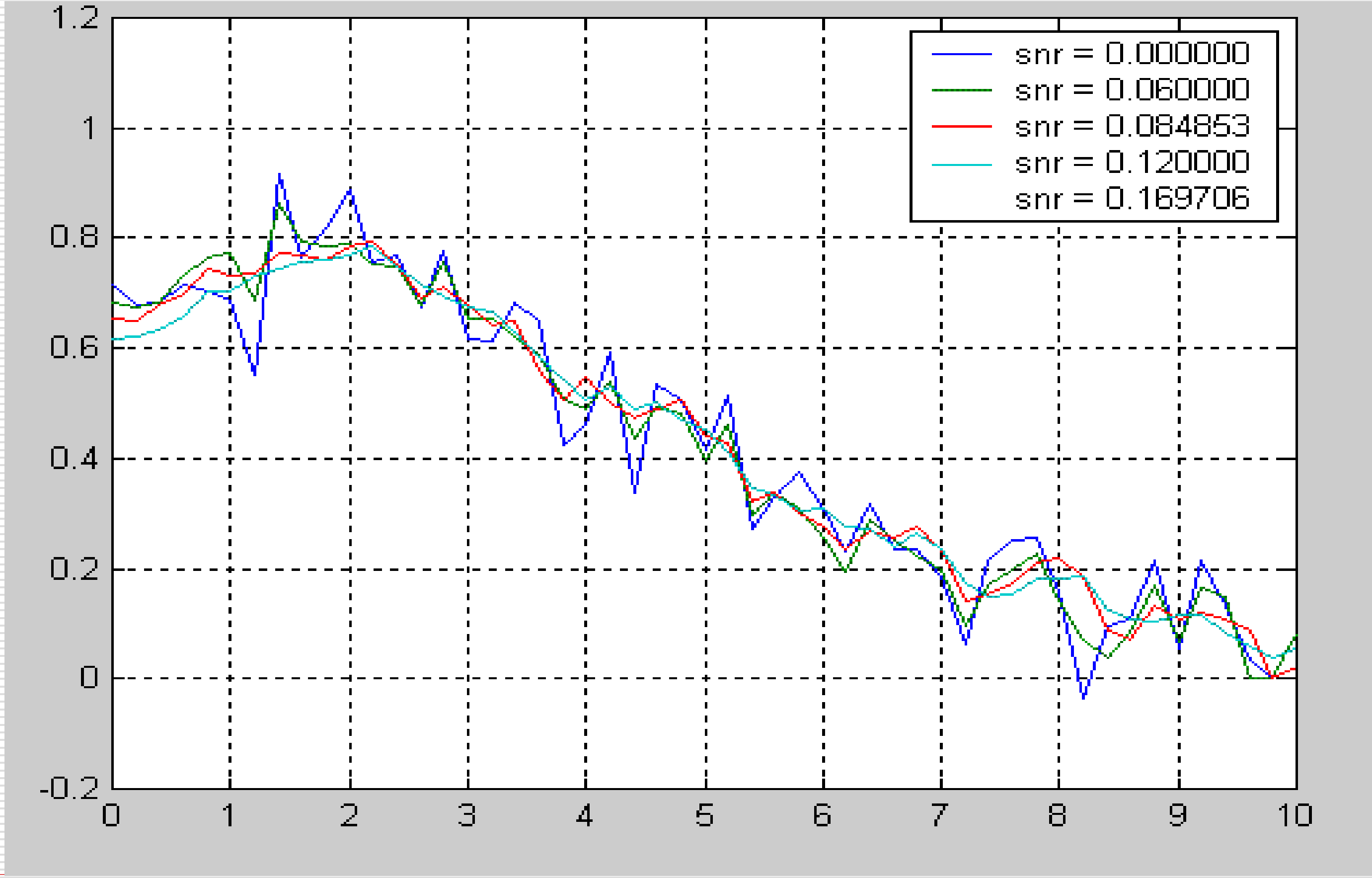
Incoherent steps: Radon transform

- using the SFDB, create the periodograms and then a time-frequency map

- for each point in the parameter space, shift and add the periodograms, in order to all the bins with the signal are added together

- the distribution of the Radon transform, in case of white noise signal, is similar to the average (or sum) of periodograms: a χ^2 with 2 N degrees of freedom, apart for a normalization.

Ratio between Hough and Radon CR (quadratic) vs threshold



Hough vs Radon

What we gain with Hough ?

• about 10 times less in computing power

• **robustness respect to non-stationarities and disturbances**

• operation with 2-bytes integers (in the simplest case)

What we lose ?

• about 12 % in sensitivity (can be cured)

• more complicate analysis

"Radon after Hough" procedure

This procedure (RaH) gives the Radon sensitivity (~12 % more) with almost the same computing power price of Hough.

It is based on doing the Radon procedure on a little percentage of points in the parameter space, selected by the Hough procedure ("Hough pre-candidates").

The computing power price is less than 10% more.

In this way, obviously, the Hough robustness is lost.

A good policy could be to follow-up both the Hough and RaH candidates.

Coherent steps

With the coherent step we partially correct the frequency shift due to the Doppler effect and to the spin-down. Then we can do longer FFTs, and so we can have a more refined time-frequency map.

This steps is done only on “candidate sources”, survived to the preceding incoherent step.

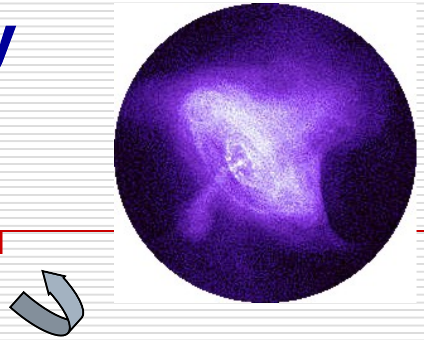
Coherent follow-up

- Extract the band containing the candidate frequency (with a width of the maximum Doppler effect plus the possible intrinsic frequency shift)
- Obtain the time-domain analytic signal for this band (it is a complex time series with low sampling time (lower than 1 Hz)
- Multiply the analytic signal samples for $e^{-j\Delta\omega_D t_i}$, where t_i is the time of the sample, and $\Delta\omega_D$ is the correction of the Doppler shift and of the spin-down.
- Create a new (partial) FFT data base now with higher length (dependent on the precision of the correction) and the relative time-frequency spectrum and peak map
- Do the Hough transform on this (new incoherent step).

A problem…

- The coherent follow-up is done on time bases of about one day or more.
- At these time scales the observed frequency is split in side bands (at distance of one sidereal day frequency and multiples)
- This is due to the rotation of the Earth and to the radiation pattern of the antenna
- This effect spreads the source power in more spectral bins, so, if it is not cured, we have lower SNR than expected

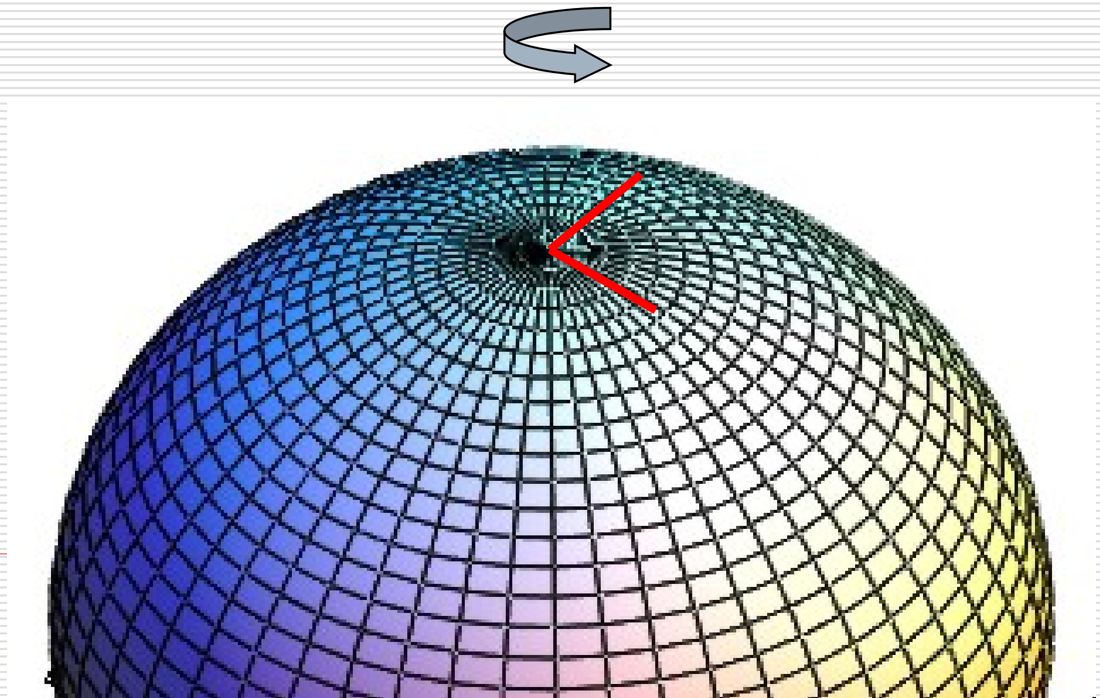
Periodic source spectroscopy



Simplified case:

Virgo is displaced to the terrestrial North Pole and the pulsar is at the celestial North Pole.

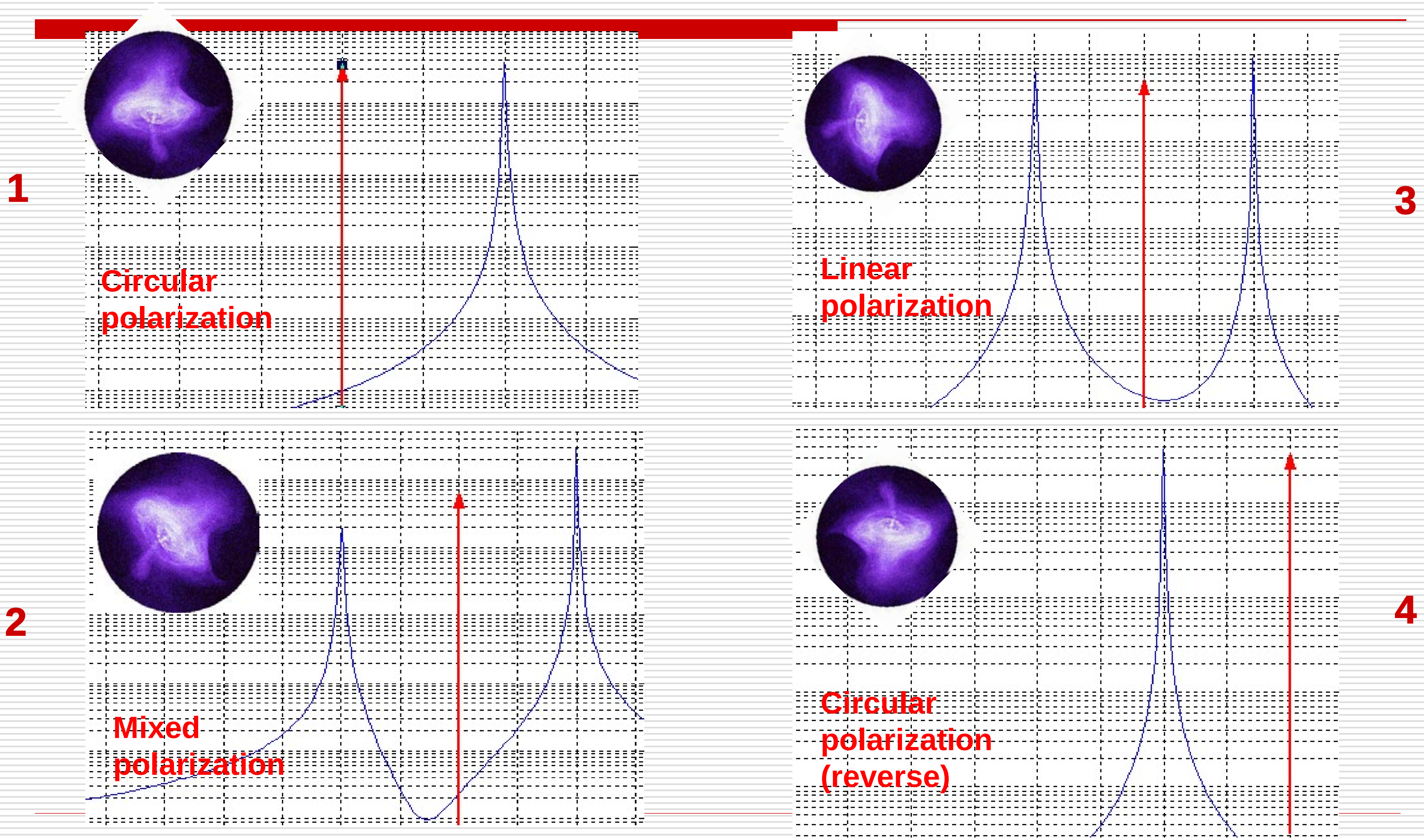
The inclination of the pulsar can be any.



Simplified case

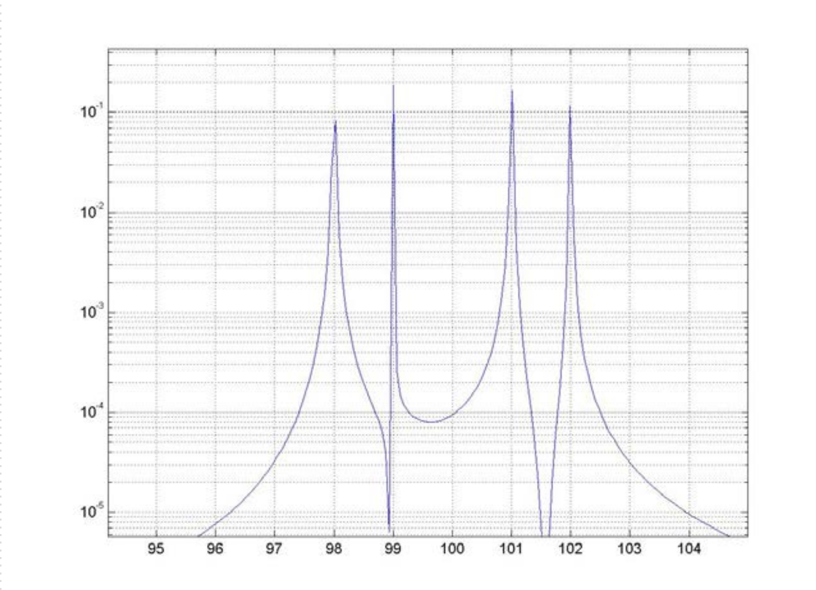
in red the original frequency

Depending on the orientation of the source axis, we have different type of polarization in the received signal.

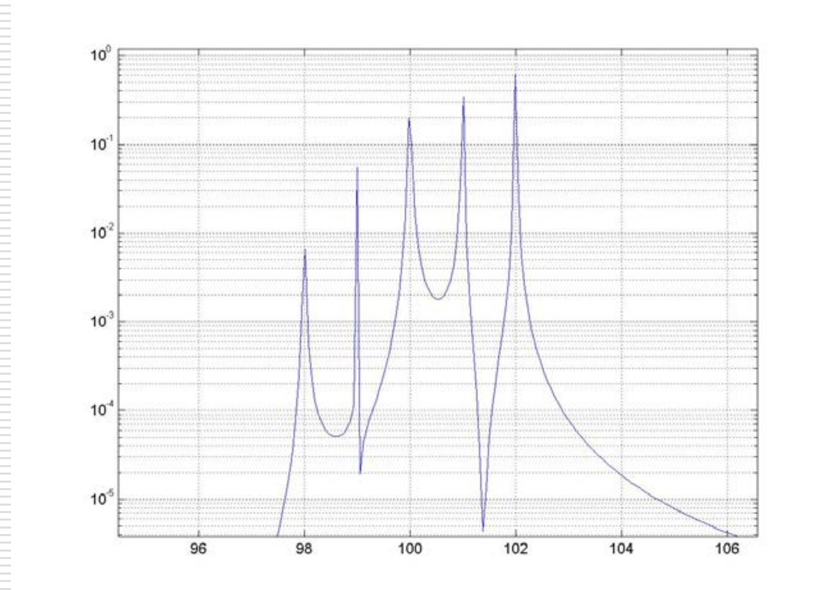


General case
(actually Virgo in Cascina and pulsar in GC)

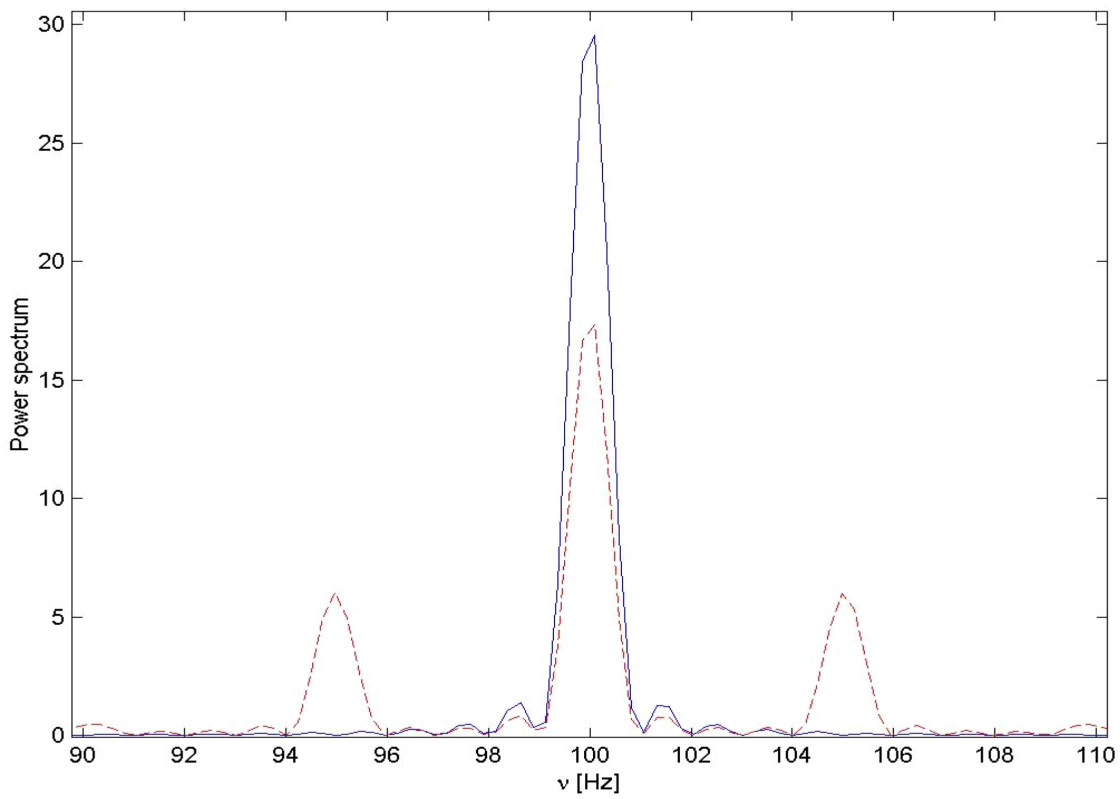
Linear polarization



Circular polarization



Wobbling triaxial star



Solution: the spectrum matched filtering

- With this procedure the power spread in different frequency bins is "collected"
- There is a matched filter for every possible value of the polarization parameters: in practice a bank of about one thousand filters

The spectral filter

Let

$$\mathbf{w}_{-2} \quad \mathbf{w}_{-1} \quad \mathbf{w}_{0} \quad \mathbf{w}_{1} \quad \mathbf{w}_{2}$$

be five numbers proportional to the power content of the five bands, with

$$\sum_{k=-2}^{2} w_k^2 = 1$$

we build the matched filter on the spectrum S(ω)

$$y(\omega) = \sum_{k=-2}^{2} w_k \cdot S(\omega + k \cdot \Omega)$$

where Ω is the sidereal angular frequency.

Hierarchical search – alternative method

- If the threshold is low, the number of candidates can be very big and the computing cost of this step, with the spectrum filtering, can be very high.
- An alternative hierarchical policy is to divide the observation period in two pieces and compute the Hough transform (and obtain candidates) for each of them, then take the coincidences between the two sets of candidates and "follow" only these one.
- Theoretically there is a loss in sensitivity of a factor 2^(1/4)~1.18, but in practice the computing burden is much lower (may be of a factor 10^6), so the the threshold can be put at lower SNR and the coherent follow-up can be done on longer time base. Also the spectral filtering can be done with no problems

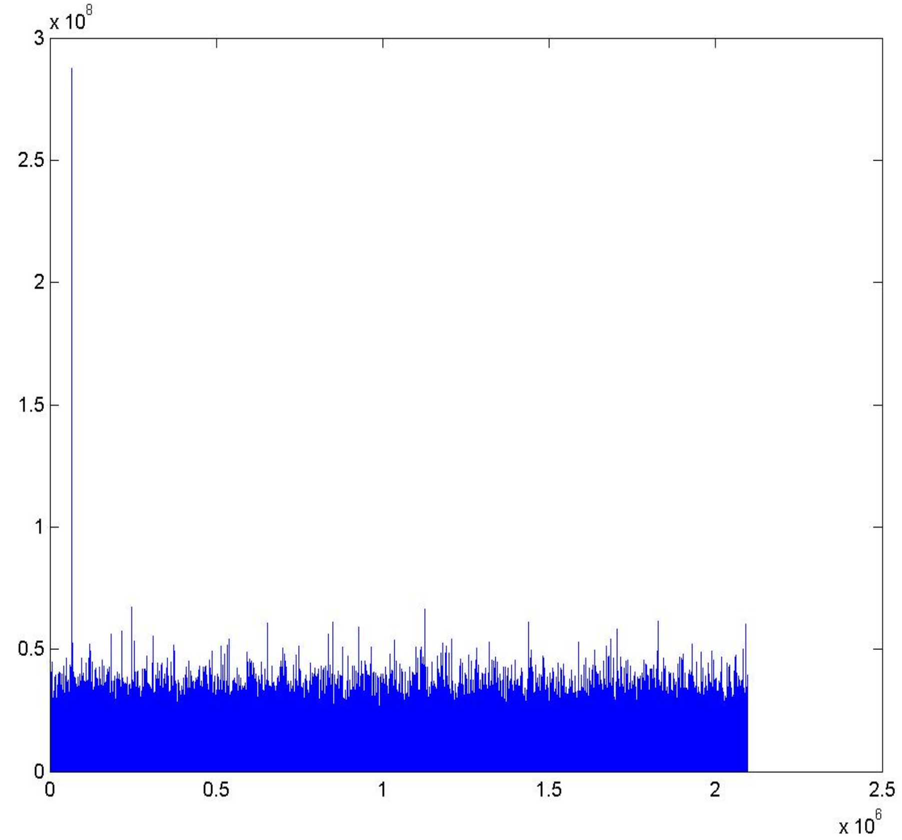
Dither effect

- The amplitude of the sinusoidal signal in the data is so low that can be 100 or more times lower than the sampling quantum (the minimal amplitude variation detectable by the analog-to-digital converter): how is it possible to detect the signal?
- It is possible because of the presence of the noise (that, in this case, has a positive effect). This effect is called **dither effect**.

Dither effect (program)

- Let us see the following matlab procedure:
  - `>> N=2^22;`
  - `>> x=(1:N)*0.1;`
  - `>> y=0.01*sin(x);` creation of a 0.01 amplitude sinusoid
  - `>> n=randn(1,N);` creation of normalized gaussian noise
  - `>> yy=round(y+n);` discretization (quantum = 1)
  - `>> sp=abs(fft(yy)).^2;` power spectrum
  - `>> plot(sp(1:N/2))`
- **Note that, discretizing only y, we obtain 0.**

Dither effect (spectrum)



The frequency peak due to the tiny signal, that was invisible because the discretization, appears.

Not always the noise is an enemy !

Other Material

The following material is complementary

It is intended to clarify some points

Number of points in the parameter space

Number of frequency bins $$N_{\nu} = \frac{T_{FFT}}{2 \cdot \Delta t}$$

Freq. bins in the Doppler band $$N_{DB} = N_{\nu} \cdot 10^{-4}$$

Sky points $$N_{sky} = 4\pi \cdot N_{DB}^2$$

Spin-down points $$N_{SD}^{(j)} = 2 \cdot N_{\nu} \cdot \left(\frac{T_{obs}}{\tau_{\min}} \right)^j$$

Total number of points $$N_{tot} = N_{\nu} \cdot N_{sky} \cdot \prod_j N_{SD}^{(j)}$$

Sensitivity

Optimal detection nominal sensitivity

$$h_{CR=1}^{(OD)} = \sqrt{\frac{4 \cdot S_h}{T_{obs}}}$$

Hierarchical method nominal sensitivity

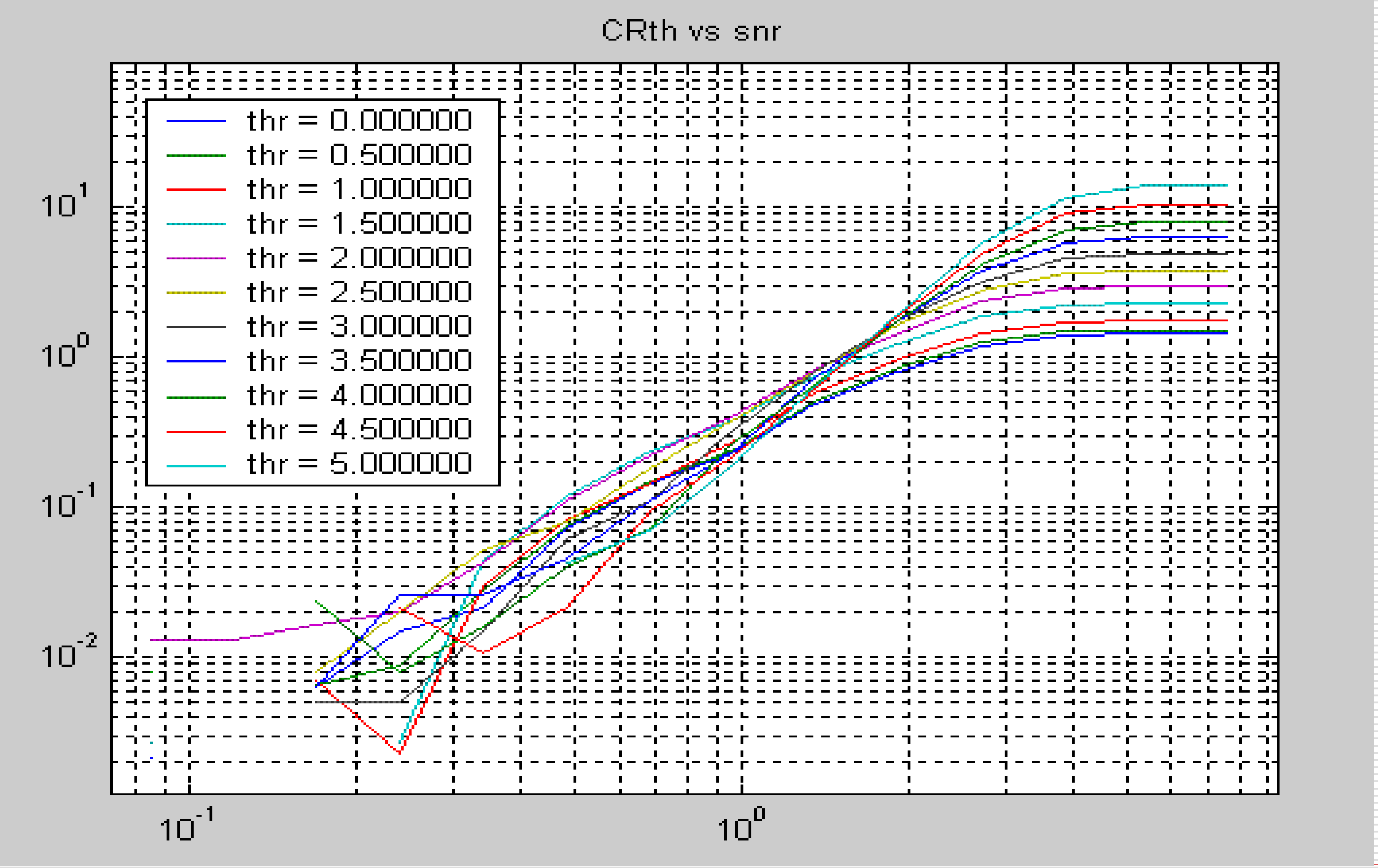
$$h_{CR=1} = h_{CR=1}^{(OD)} \cdot \sqrt[4]{\frac{T_{obs}}{T_{FFT}}}$$

Hierarchical search results

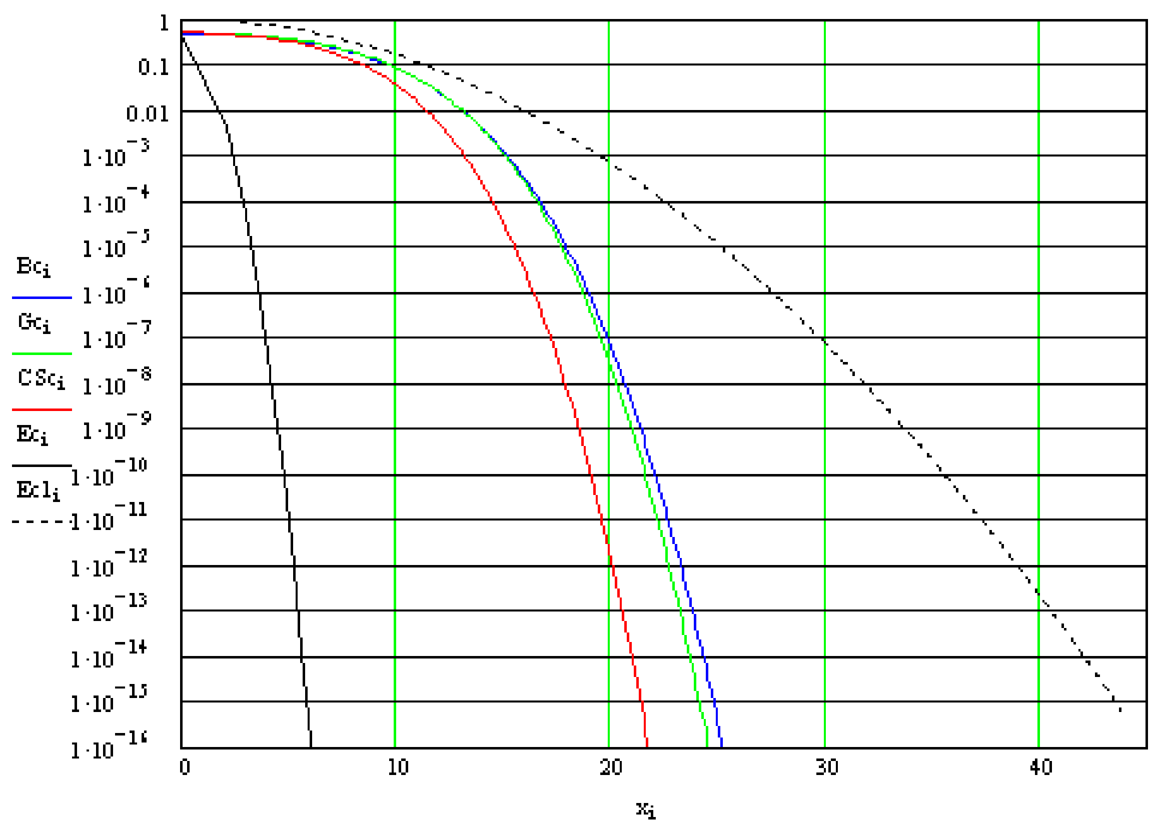
SFDB band	Band 1	Band 2	Band 3	Band 4
Doppler bandwidth (Hz)	0.2	0.05	0.0125	0.0032
Angular resolution in the sky (rad)	0.0038	0.0038	7.6294E-03	1.5259E-02
Number of pixels in the sky	8.6355E+05	8.6355E+05	2.1589E+05	5.3972E+04
Number of independent frequencies	1.5729E+06	1.5729E+06	7.8643E+05	3.9322E+05
Spin down parameters (only order 1)	140	140	70	35
Tot. number of parameters (one freq)	1.207E8	1.207E8	1.509E7	1.886E6
Number of operations for one peak	6.5884E+03	6.5884E+03	3.2942E+03	1.6471E+03
Total number of operations	6.348E+18	1.587E+18	4.959E+16	1.55E+15
Comp. Pow. for the 1st step (GFlops)	1030	257	8.0	0.251
Overall computing power (Gflops)	2000	500	15	0.5
Nominal sensitivity	6.17E-26	4.36E-26	3.67E-26	3.08E-26
Practical sensitivity	1.23E-25	8.72E-26	7.33E-26	6.17E-26

Minimum decay time considered is 10^4 years

Hough transform vs SNR



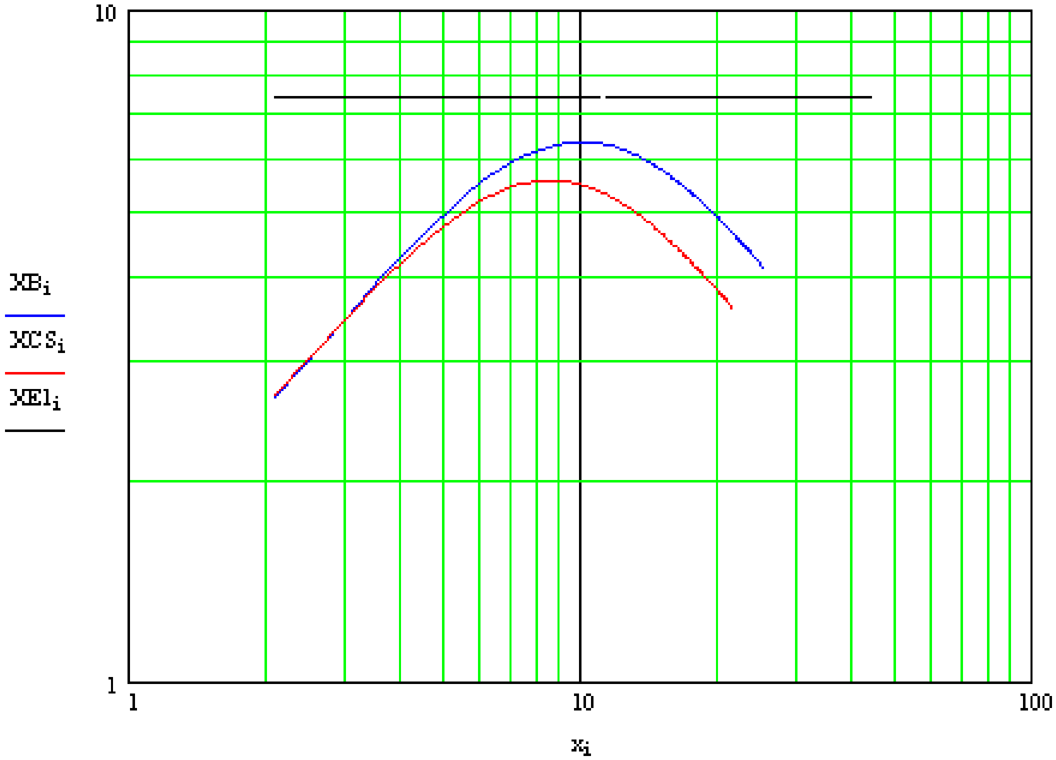
Noise distributions - linear



The black line is the noise distribution for the optimum detection, the red one is for the hierarchical procedure (hp) with Radon, the blue and green are for hp with Hough (the green is the gaussian approximation) and the dotted line is for a short FFT.

There were 3000 pieces.

Loss respect to the optimum



In this plot there is the SNR loss (respect to the optimum detectipon) for the hierarchical procedure with Hough (blue) and Radon (red) and a short FFT (black).

In abscissa there is the SNR..

Tuning a hierarchical search

The fundamental points are:

• the sensitivity is proportional to $\sqrt[4]{T_{FFT}}$

• the computing power for the incoherent step is proportional to T_{FFT}^3

• the computing power for the coherent step is proportional to $\log T_{FFT}$, but it is also proportional to the number of candidates that we let to survive.

What is a candidate source ?

The result of an analysis is a list of candidates (for example, 10^6 candidates).

Each candidate has a set of parameters:

- the frequency at a certain epoch
- the position in the sky
- 2~3 spin-down parameters

Detecting periodic sources

The main point is that a periodic source is permanent. So one can check the “reality” of a source candidate with the same antenna (or with another of comparable sensitivity) just doing other observations.

So we search for “coincidences” between candidates in different periods.

The probability to have by chance a coincidence between two sets of candidates in two 4-months periods is of the order of 10^{-20}.

Coincidences

In case of non-ideal noise, the preceding f.a. probabilities can be not reliable, nevertheless there are some methods to validate the survived candidates. One is the coincidence method.

If $\mathbf{n_1}$ and $\mathbf{n_2}$ candidates survive in two different four-months periods (for example $\mathbf{n_1 = n_2 = 10}$, at the third step, where the number of points in the parameter space $\mathbf{N_P}$ is about **6.e24**) , we can seek for coincidences between the two sets, i.e. check if there are some with equal (or similar) parameters.

The expected number of coincidences (or the probability of a coincidence) is

$$n_{COIN} = \frac{n_1 \cdot n_2}{N_P}$$

with the values of our example, $\mathbf{n_{COIN}}$**=6.e-22 .**

False alarm probability

In the case of the periodic source search with the hierarchical method, the false alarm probability is normally embarrassingly low. This for two reasons:

- the hierarchical procedure produces at the first step a high number of candidates and for them the f.a. probability is practically 1, but already at the second step the candidates disappear and it plunges at very low levels.

- if some false candidates survive, the coincidence with the survived candidates (with the same parameters) in other periods or in other antennas lower the f.a. probability at levels of absolute impossibility.

Computing Hough f.a. probability

Let us start from a random peak map. Let **p** (~0.1) be the density of the peaks on the map. The value k of a pixel of the Hough map follows a binomial distribution

$$\binom{M}{k} p^k (1-p)^{M-k}$$

where **M** is the number of spectra.

If there is a weak signal, the expected value of k is enhanced by an amount proportional to the square of the amplitude of the signal. So if there is a certain (linear) **SNR** at a certain step, at the following one, with a 16 times longer $\mathbf{T_{FFT}}$, there is a **CR** four times higher.

"Old" scheme of the detection

T_{OBS} = 4 months T_{FFT} = 3355 s

step	T_{FFT}	N points	SNR (linear)	CR	Normal probability	Candidates
1	~1 h	1.5 e15	2	4	3.1 e-5	5 e10
2	15 h	9.8 e19	4	16	~1 e-55	1 e-35
3	10 d	6.4 e24	8	64	…	…
4	~4 m	4.2 e29	~16	~256	…	…

Sensitivity

The signal detectable with a CR of 4 (5.E10 candidates in the band from 156 to 625 Hz) is given by

$$h_{CR=4} \approx 2\frac{2S_h}{\sqrt[4]{T_{OBS} \cdot T_{FFT}}} = 2.8 \cdot 10^{-25}$$

with T_{OBS}=4 months, T_{FFT}=3355 s, S_h=3E-23 $Hz^{-1/2}$.