CW methods and searches

Part I

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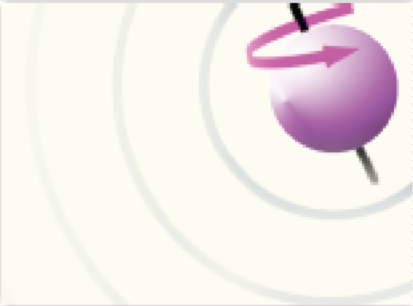
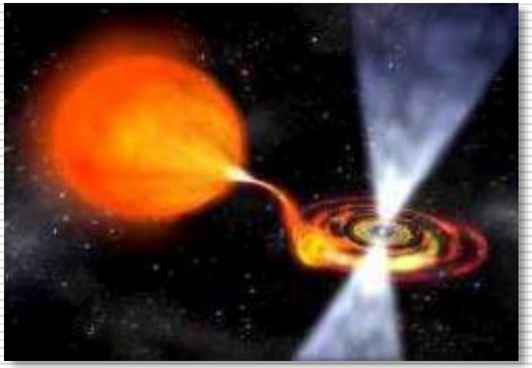
Outline

- Detection of periodic signals: power spectrum
- Quasi-periodic signals: spin-down, Doppler effect, radiation pattern
- Peculiarity of continuous waves
- Resampling
- Complex signals and heterodyne
- Computing power
- Hierarchical procedure
- Dither effect

Continuous waves (CW)



- We classify as continuous those GW signals with duration much longer than the typical observation time of detectors.
- CW are typically emitted by sources with a mass quadrupole moment varying in time in a quasi-periodical way.
- For Earth-bound detectors the most interesting sources of CW involve deformed neutron stars (NS), isolated or in binary systems.
- We know that potential sources of CW exist: 2400+ NS are observed (mostly pulsars) and O(109) are expected to exist in the Galaxy.



Signal characterization

- **Shape:** sinusoidal, possibly two harmonics.
- **Location:** our galaxy, more probable near the center or in globular clusters; nearest (and more detectable) sources are isotropic; **sometimes it is known, often not (blind search).**
- **Frequency:** down, limited by the antenna sensitivity; up to 1~2 kHz; **sometimes it is known , often not (blind search).**
- **Amplitude:**

$$h_0 = 1.05 \cdot 10^{-27} \left(\frac{I_3}{10^{38} kg \cdot m^2} \right) \left(\frac{10kpc}{r} \right) \left(\frac{\nu}{100Hz} \right)^2 \left(\frac{\varepsilon}{10^{-6}} \right)$$

I_3 is the principal moment of inertia along the rotation axis, ε is the ellipticity $(I_2-I_1)/I_3$

Simple periodic signal

The simplest way to describe a CW signal is simply

$$h(t) = A \cdot \sin(\omega_0 t) = \frac{A}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

If we have a chunk of length $\mathbf{T_0}$ of this signal embedded in (white Gaussian) noise, we know that the better detection algorithm is the matched filter, i.e. the convolution between the data x(t) (signal+noise) and the signal shape f(t)

$$y(t) = \int_0^{T_0} x(t)\ f(t-\tau)\, d\tau$$

(**f(t)** could be just $\mathbf{\sin(\omega_0 t)}$). This produce the output y(t) with the maximum value of the SNR.

Fourier transform

If the angular frequency is unknown, the ideal way to detect the signal is the estimation of the power spectrum, that normally is obtained taking the square of the absolute value of the Fourier transform:

$$S(\omega) = \frac{1}{T_0} \left| \int_0^{T_0} x(t) \cdot e^{-j\omega t} \, dt \right|^2$$

This estimation is called **periodogram**.

The resolution in frequency f *(=ω/2π)* is $\Delta f = 1/T_0$. The power spectrum describes the frequency distribution of the noise power, that is independent from T_0 . The power of the signal goes all in a band that has a width inversely proportional to T_0 . So the spectral (quadratic) signal to noise is proportional to T_0 .

Power spectrum by FFT periodogram

If the frequency (and the phase) of the signal is not known, the better way to detect a periodic signal is by the **estimate of the power spectrum**. This can be obtained by a **periodogram**, i.e. the square modulus of the Fourier transform of the data.

Remember that the power spectrum is, by definition, the Fourier transform of the **autocorrelation** $R_{xx}(\tau)=E[x(t)x(t+\tau)]$.

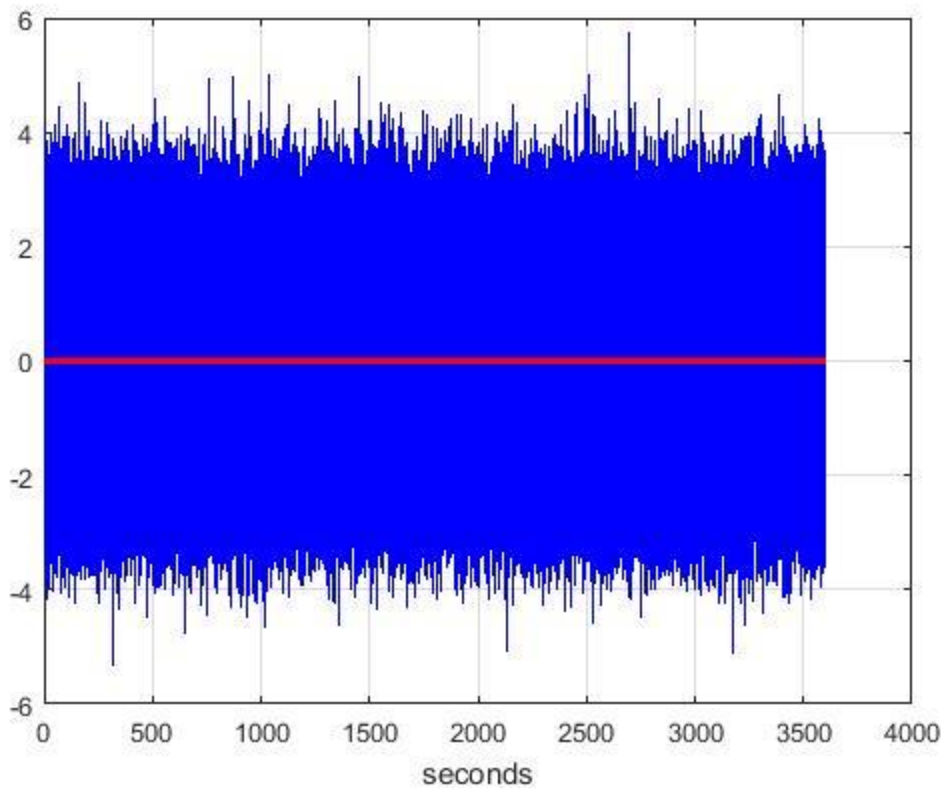
PS with FFT periodogram is like a set of lock-in or matched filters at n frequencies.

An efficient algorithm to compute the (discrete) Fourier transform of the sampled data is the *Fast Fourier Transform* (FFT). The number of floating point operations (FLOP) needed to compute an FFT of length n (that should be a power of 2) is about

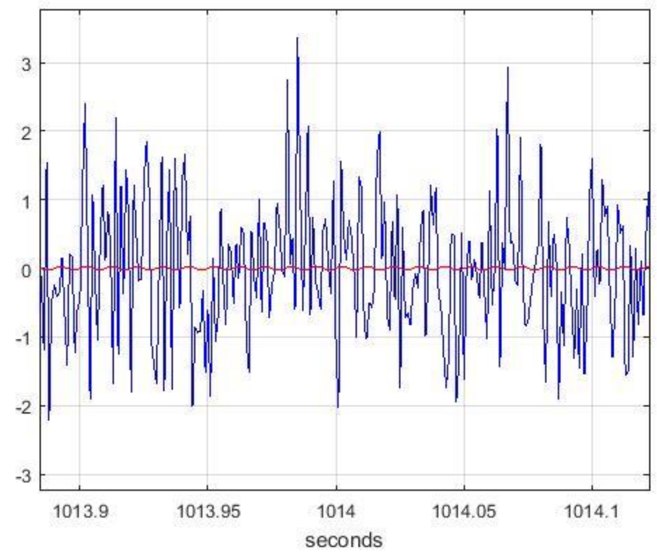
$$5 * n * \log_2(n)$$

instead of something proportional to n^2.

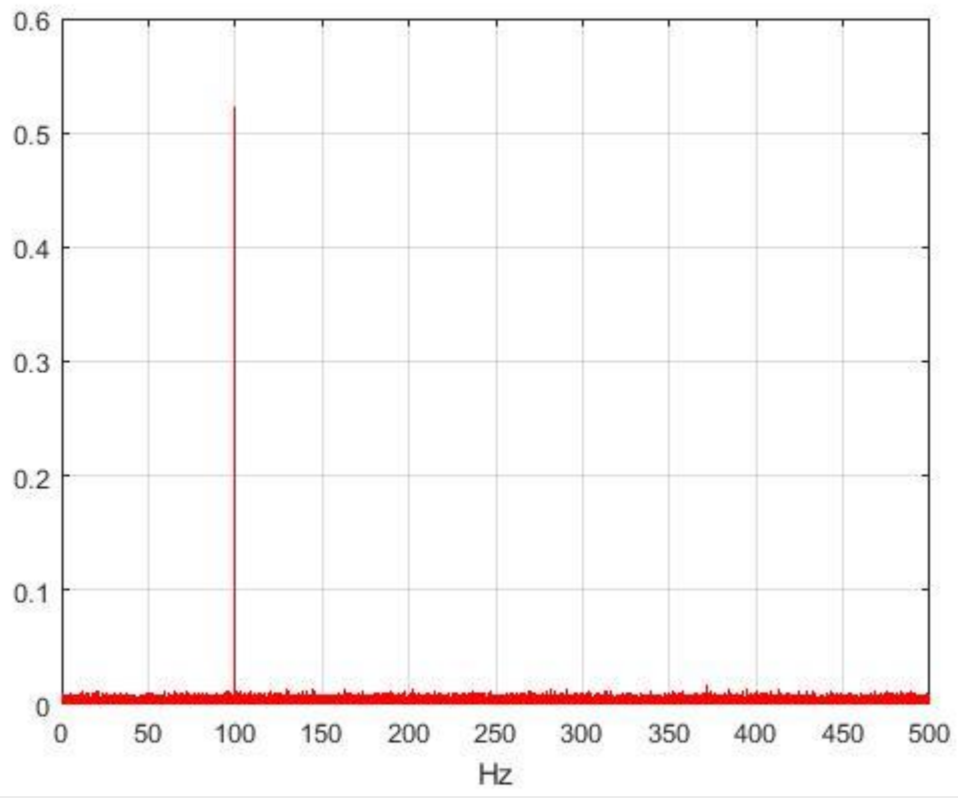
Example

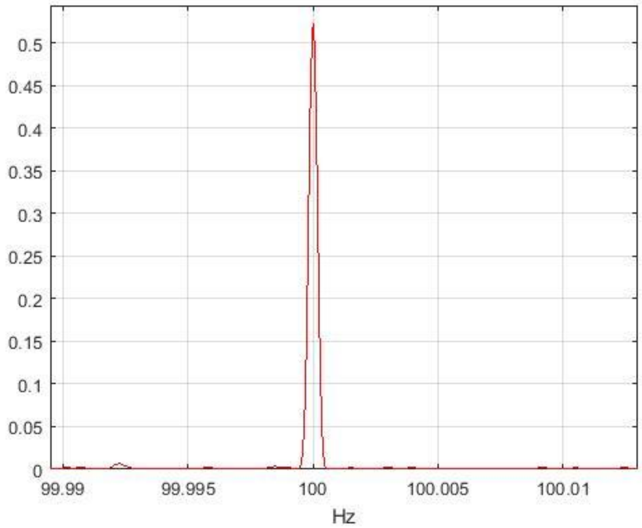


Frequency 100 Hz
Noise σ = 1
Signal A = 0.03
Power SNR ~ 0.001



Power spectrum





Power spectrum by FFT periodogram (some details)

Discrete Fourier transform

$$X_k = \sum_{i=0}^{n-1} x_i \cdot e^{j \cdot \frac{k \cdot i}{n}}$$

Frequency resolution

(if the signal power goes all in a single bin, the noise power in the bin is proportional to the bin width)

$$\Delta \nu = \frac{1}{T_{obs}}$$

Signal-to-noise ratio (linear)

$$SNR_{PS} = \frac{h_0 \cdot \sqrt{T_{obs}}}{2 \cdot H_n(\omega_0)} \approx 1.6 \cdot \left(\frac{h_0}{10^{-26}} \right) \left(\frac{H_n(\omega_0)}{10^{-23}\, Hz^{-1/2}} \right)^{-1} \left(\frac{T_{obs}}{10^7\, s} \right)^{\frac{1}{2}}$$

$\sqrt{2}$ less than the SNR of the matched filter

Power spectrum as the mean of periodograms

The distribution of the amplitude of the bins of the periodogram of a chunk of white gaussian noise is exponential. It remains exactly the same increasing the length of the periodogram, and the same is obviously for the mean and the variance.

To reduce the variance of the noise spectrum, one way is by dividing the chunk of data in **N** pieces, take the periodograms of each piece and then make the average.

In this case both the variance and the signal is reduced and the (linear) SNR is reduced by a factor $\sqrt[4]{N}$.

The distribution is a χ^2 with 2N degrees of freedom.

Sampling

Obviously we work with sampled data (with sampling time **Δt**) and we can compute the Fourier transform with an **FFT** (fast Fourier transform) algorithm.

In case of **real** sampled data the transform has a bandwidth $B=1/2T_0$ and the estimation of the power spectrum is symmetric around 0.

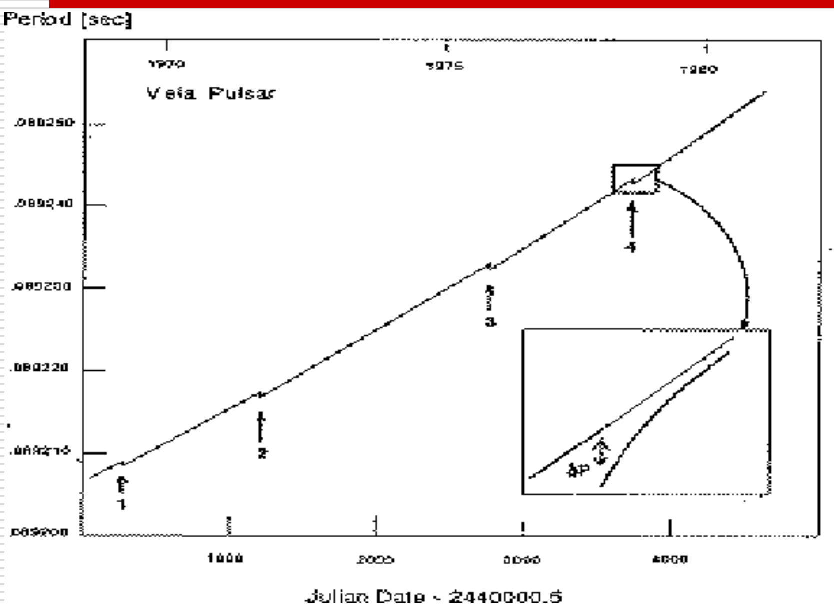
In case of **complex** data, this symmetry is not present and the bandwidth is $B=1/T_0$.

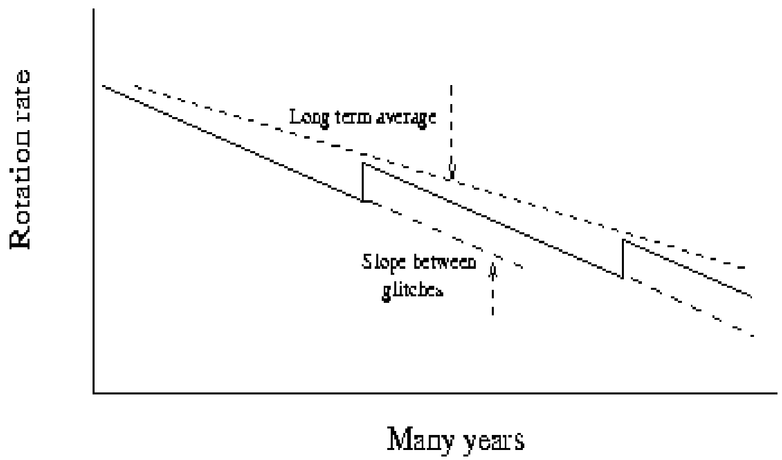
Problem: the frequency is not constant !

If things were as in this simple way, we should do the largest FFT and obtain the best sensitivity, but in reality the frequency is not constant. This because:

- there is a **spin-down** due to the loss of energy (in some cases a spin-up could be present)
- the source can be in a binary star system, and so an **"intrinsic" source Doppler shift**
- due to the orbital and rotational motion of the Earth (and of the antenna), there is a **detector Doppler shift**, dependent on the direction of the source
- Because of the variation of the direction of the source in the frame of the detector, we have a **sidereal day variation of the phase and amplitude** of the detected signal
- sometimes a **glitch** (sudden variation of the rotational frequency of the star) can appear
- a small **phase noise** can be present

Glitches





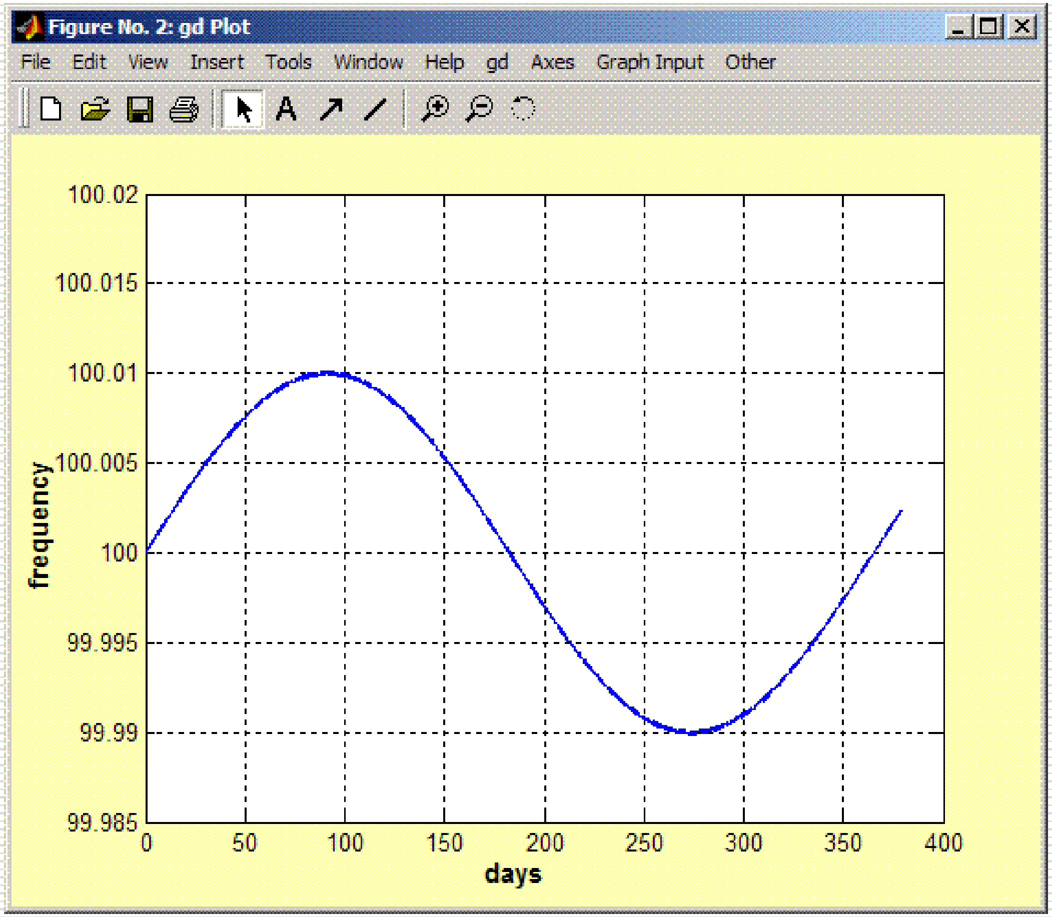
In the first figure there is the period variation of the Vela Pulsar during about 12 years.

In the second figure there is a more schematic view (frequency on ordinates).

The frequency of glitches depends on the age of the star (younger stars have more glitches) and is not a general feature.

Glitches are related to star-quakes.

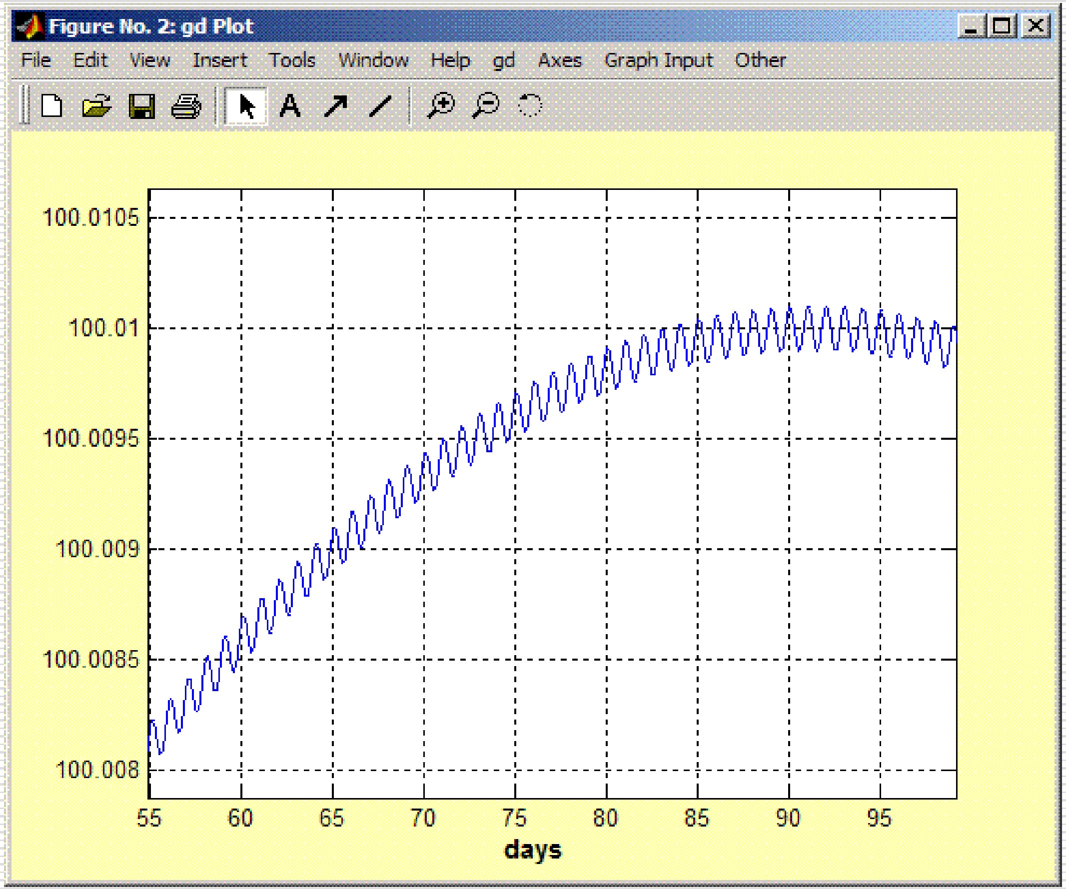
Detector Doppler effect



The Doppler variation of the frequency in a period of one year for a low (ecliptical) latitude source.

The original frequency is 100 Hz and the maximum variation fraction is of the order of 0.0001

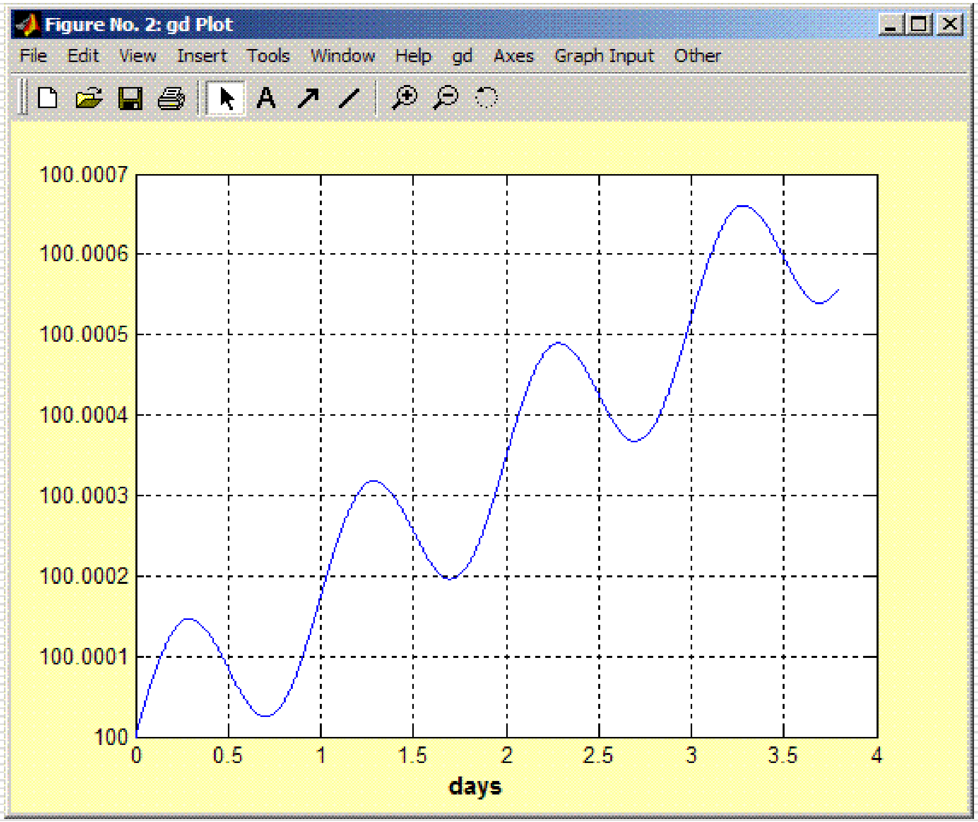
Doppler effect - zoom



Zoom of the preceding figure. Note the daily variations.

Very roughly the Doppler effect can be seen as the sum of two "epicycles" (Ptolemaic view)

Doppler effect (zoom)



Frequency variation on about 4 days.

Note that the problem is not in the presence of the Doppler shift, but in the time variation of the Doppler shift. So the effect of the rotation is more relevant of that of the orbital motion. The rotation epicycle is dominant.

Peculiarity of the periodic sources

The variation of the frequency of a periodic source is a big problem, but is an important instrument

- to have physical information on the source, e.g. the sky direction, that can be known with high precision
- to exclude disturbance and artifacts: in fact the Earth Doppler effect is strong signature for the real signals.

The periodic sources are the only type of gravitational signal that can be detected by a single gravitational antenna with certainty (if there is enough sensitivity to include the source among the candidates, the **false alarm probability** can be reduced at any level of practical interest).

Once one detects a periodic source, it remains there to be confirmed and studied by others. It is not only a detection, it is a discovery.

Frequency correction

If the frequency variation is known or can be modeled by a set of templates, we can correct the frequency variation and reduce the problem to the simple one.

Normally this can be done up to a certain degree.

The frequency correction can be achieved with a variety of methods using two base techniques:

- the **resampling**
- the **heterodyne**

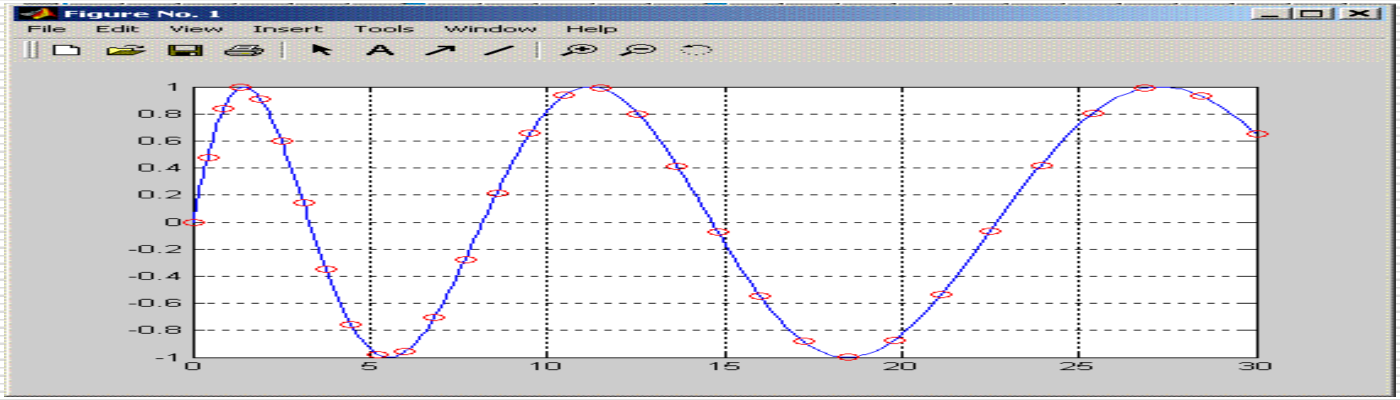
Solution: re-sampling procedure

Because of the frequency variation, the energy of the wave doesn't go in a single bin, so the SNR is highly reduced.

A solution to the problem of the varying frequency is to use a non-uniform sampling of the received data: if the sampling frequency is proportional to the (varying) received frequency, the samples, seen as uniform, represent a constant frequency sinusoid and the energy goes only in one bin of their FFT.

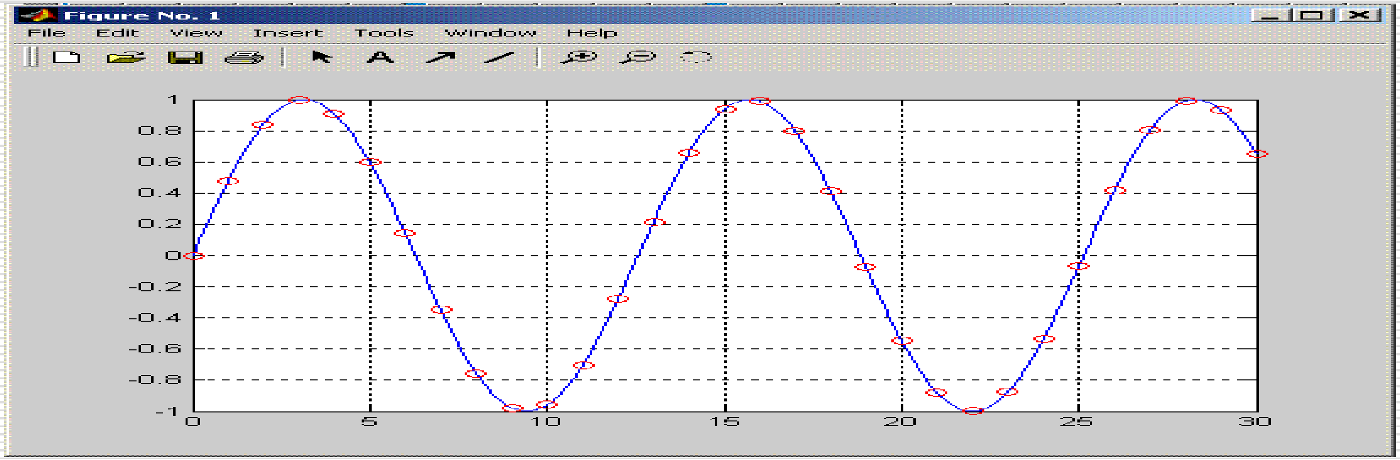
Every point of the sky (and every spin-down or spin-up behavior) needs a particular re-sampling and FFT.

Resampling



Original data:

The frequency is varying, we sample non-uniformly (about 13 samples per period).



The non-uniform samples, seen as uniform, give a perfect sinusoid and the periodogram of the samples has a single "excited" bin.

Solution: the sub-heterodyne and the complex signal

To simplify the problem it is useful to convert the real two sided data to complex single sided. This can be accomplished by the use of the Fourier transform.

The small frequency variation signal can be written, in the complex form, as

$$h(t) = \exp\left(j \cdot \left(\omega_0 t + \Delta\varphi(t) \right) \right)$$

If we know, or hypothesize with a set of templates, φ(t), we can correct the signal multiplying h(t0 for exp(-jΔφ(t)) and we obtain the simple signal

$$h(t) = \exp\left(j \cdot \omega_0 \cdot t \right)$$

Use of sub-heterodyne

To correct the variation of frequency due to the Doppler effect or to the spin-down, we should hypothesize an enormous number of separate points in the parameter space, for each of them we should do a different correction.

This problem will lead to the use of hierarchical procedures.

Interaction with the antenna: the 5-vector formalism

After the frequency correction, the signal can be described as a complex signal

$$\mathbf{h}(t) = h_0 \cdot \left(\mathbf{e}_\oplus \cdot \kappa_+ + \mathbf{e}_\otimes \cdot \kappa_\times \exp\left(j\varphi\right)\right) \cdot \exp\left(j \cdot \left(\omega_0 t + \alpha\right)\right)$$

where the tensors are in bold Arial font, h_0 is the wave amplitude and κ_+ and κ_x are positive constant with $\kappa_+^2 + \kappa_\times^2 = 1$.

The response of the antenna to a gravitational wave depends on the relative direction of the source and has the form of the linear combination of five complex exponentials

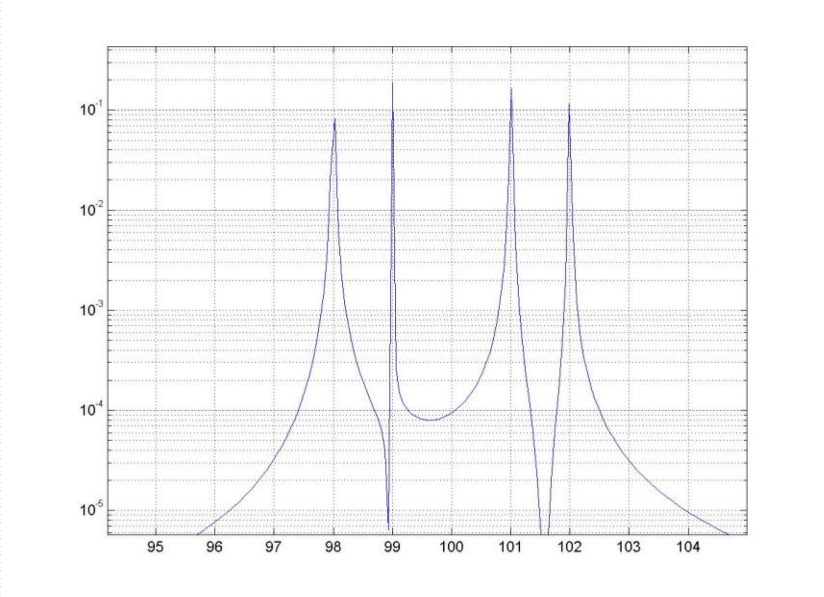
$$r(t) = \sum_{k=-2}^{2} v_k \cdot \exp\left(j \cdot \left(\omega_0 t + \varphi_0\right)\right) \cdot \exp\left(j \cdot k \cdot \Omega_{sid} t\right)$$

where the **v_k are 5 complex constants** and Ω_{sid} is the Earth sidereal angular frequency.

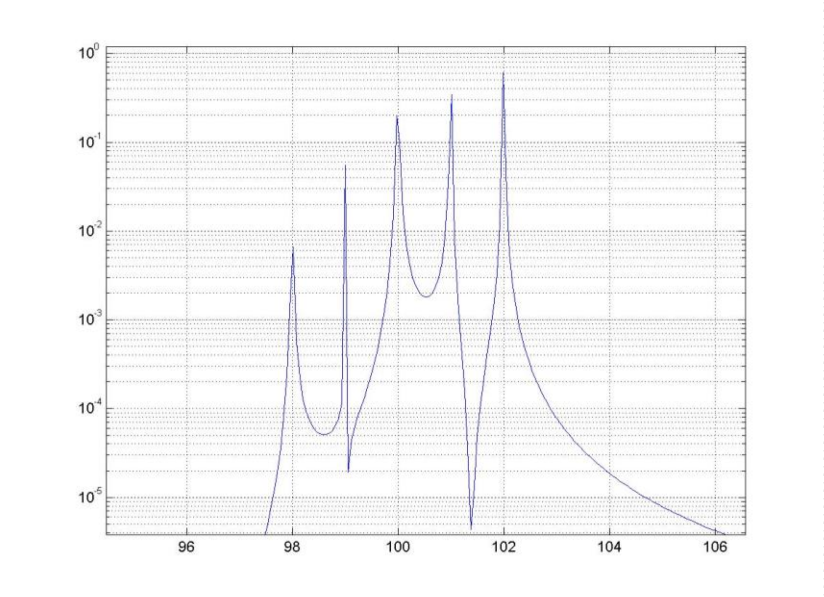
So, if we have enough frequency resolution in the spectrum, we see 5 peaks.

Some cases
(actually Virgo antenna and pulsar in Galactic Center)

Linear polarization



Circular polarization



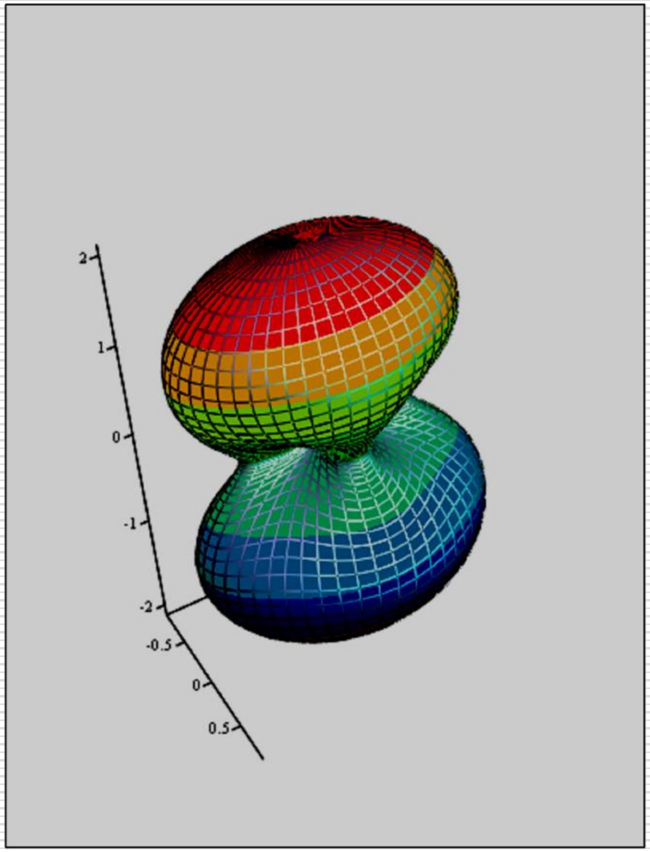
Note that in the first case the complex constant $v_0 = 0$

The radiation pattern

The interference of the five frequencies give rise to the radiation pattern of the antenna.

From the estimation of the five complex numbers, we can estimate the polarization parameters, the amplitude and the phase of the wave.

Because the degree of freedom of the 5 complex numbers are 10 and the gravitational waves has only 4 d.o.f., we can use the estimation of that 5 as a filter against false alarms.



(X4, Y4, Z4)

Optimal detection

	1 month	4 months	1 year
FFT length (number of points)	2.6E+09	1.0E+10	3.1E+10
Sky points	2.1E+11	3.4E+12	3.1E+13
Spin-down points (1st)	2.2E+04	3.5E+05	3.1E+06
Spin-down points (2nd)	1.0E+00	1.1E+01	3.2E+02
Freq. points (500 Hz)	1.3E+09	5.0E+09	1.6E+10
Total points	6.0E+24	6.5E+28	4.8E+32
Comp. power (Tflops)	1.0E+12	1.5E+16	3.6E+19
Sensitivity (nominal)	1.2E-26	6.0E-27	3.6E-27
(background 10^-23*Hz^-0.5)			
Sens. for 10^9 candidates	7.4E-26	4.1E-26	2.7E-26

It is supposed a 2 kHz sampling frequency. For the computation power, an highly optimistic estimation is done and it is not considered the computation power needed by the re-sampling procedure. The decay time (spindown) is taken higher than 10^4 years.

Introduction to the hierarchical search

- Because the "optimal detection" cannot be done in practice, we have proposed the use of a sub-optimal method, based on alternating "incoherent" and "coherent" steps
- The first incoherent step consist normally of Hough or Radon transform based on the collection of short FFT periodograms. From this step we "produce" candidates of possible sources
- Then, with a coherent step, we "zoom" on the candidates, refining the search
- Then a new incoherent step can be done, and so on, until the full sensitivity is reached

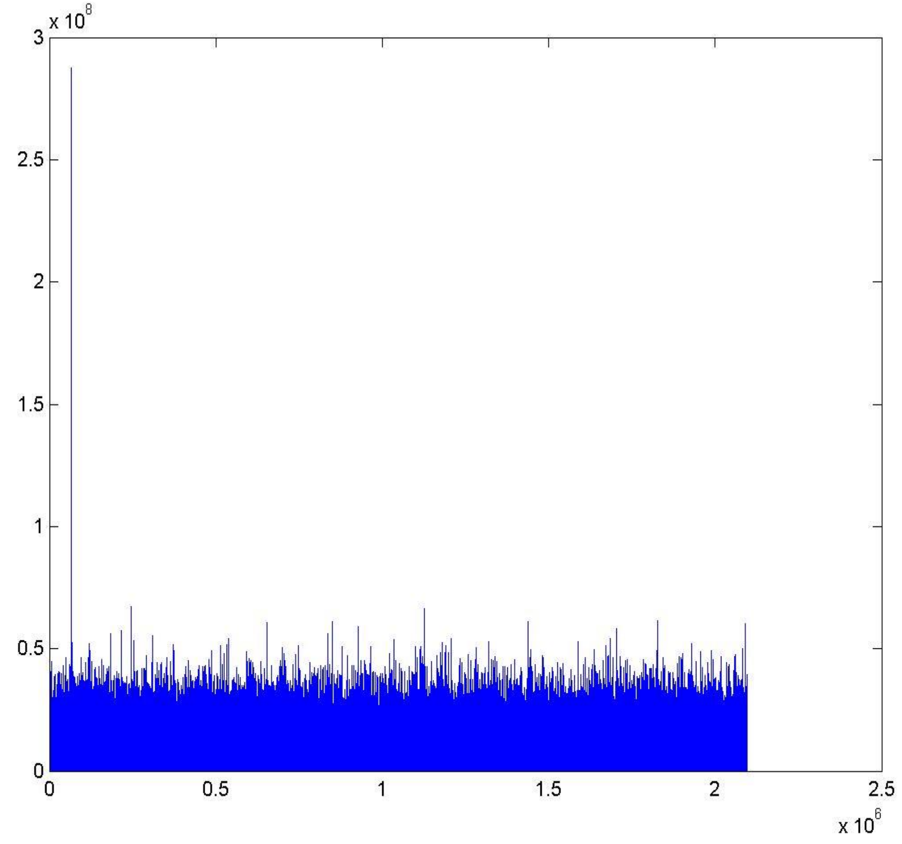
Dither effect

- The amplitude of the sinusoidal signal in the data is so low that can be 100 or more times lower than the sampling quantum (the minimal amplitude variation detectable by the analog-to-digital converter): how is it possible to detect the signal?
- It is possible because of the presence of the noise (that, in this case, has a positive effect). This effect is called **dither effect**.

Dither effect (program)

- Let us see the following matlab procedure:
  - >> N=2^22;
  - >> x=(1:N)*0.1;
  - >> y=0.01*sin(x); creation of a 0.01 amplitude sinusoid
  - >> n=randn(1,N); creation of normalized gaussian noise
  - >> yy=round(y+n); discretization (quantum = 1)
  - >> sp=abs(fft(yy)).^2; power spectrum
  - >> plot(sp(1:N/2))
- **Note that, discretizing only y, we obtain 0.**

Dither effect (spectrum)



The frequency peak due to the tiny signal, that was invisible because the discretization, appears.

Not always the noise is an enemy !

Other Material

The following material is complementary

It is intended to clarify some points

Number of points in the parameter space

Number of frequency bins $$N_\nu = \frac{T_{FFT}}{2 \cdot \Delta t}$$

Freq. bins in the Doppler band $$N_{DB} = N_\nu \cdot 10^{-4}$$

Sky points $$N_{sky} = 4\pi \cdot N_{DB}^2$$

Spin-down points $$N_{SD}^{(j)} = 2 \cdot N_\nu \cdot \left(\frac{T_{obs}}{\tau_{\min}} \right)^j$$

Total number of points $$N_{tot} = N_\nu \cdot N_{sky} \cdot \prod_j N_{SD}^{(j)}$$

Sensitivity

Optimal detection nominal sensitivity

$$h_{CR=1}^{(OD)} = \sqrt{\frac{4 \cdot S_h}{T_{obs}}}$$

Hierarchical method nominal sensitivity

$$h_{CR=1} = h_{CR=1}^{(OD)} \cdot \sqrt[4]{\frac{T_{obs}}{T_{FFT}}}$$

FFT and Doppler effect

$$T_{FFT}^{(max)} = \frac{1}{\Omega_0}\sqrt{\frac{c}{2 \cdot \nu_0 \cdot R_0}}$$

	Period	Radius	$R_0 \cdot \Omega_0$ *velocity*	$R_0 \cdot \Omega_0^2$	Max Doppler %	Tmax a 2kHz	N Doppler bin a 2kHz a Tmax"min"
	s	m				s	
Rotation (equator)	86160	6.38E+06	4.65E+02	0.033928	3.10E-06	1.49E+03	11
Rotation (43° N)	86160	4.66E+06	3.40E+02	**0.024781**	2.27E-06	**1.74E+03**	**8**
Earth orbit	31557600	1.49E+11	**2.97E+04**	0.005906	**1.98E-04**	3.56E+03	690
Moon	2505600	4.75E+06	1.19E+01	2.99E-05	7.95E-08	5.01E+04	0
Jupiter	3.72E+08	7.39E+08	1.25E+01	2.10E-07	8.32E-08	5.97E+05	0
Saturn	9.28E+08	4.06E+08	2.75E+00	1.86E-08	1.83E-08	2.01E+06	0

Backup (old)

Short periodograms and short FFT data base

- The basis of the hierarchical search method is the "short FFT data base"
- It is used for producing the periodograms for the incoherent steps and the data for the coherent step
- How long should be a "short FFT" ?

Short periodograms and short FFT data base (continued)

What is the maximum time length of an FFT such that a Doppler shifted sinusoidal signal remains in a single bin ? (Note that the variation of the frequency increases with this time and the bin width decreases with it)

The answer is

$$T_{\max} = T_E \cdot \sqrt{\frac{c}{4\pi^2 R_E \nu_G}} \approx \frac{1.1 \cdot 10^5}{\sqrt{\nu_G}} \; s$$

where T_E and R_E are the period and the radius of the "rotation epicycle" and ν_G is the maximum frequency of interest of the FFT.

Short periodograms and short FFT data base (continued)

As we will see, we will implement an algorithm that starts from a collection of short FFTs (the SFDB, short FFT data base).

Because we want to explore a large frequency band (from ~10 Hz up to ~2000 Hz), the choice of a single FFT time duration is not good because, as we saw,

$$T_{\max} \propto \nu_G^{-\frac{1}{2}}$$

so we propose to use 4 different SFDB bands.

Hough transform

Another way to deal with the changing frequency signal, starting from a collection of short length periodograms, is the use of the Hough transform (see **P.V.C. Hough, "Methods and means for recognizing complex patterns", U.S. Patent 3 069 654, Dec 1962**)

Linear Hough transform

Suppose to have an image of one particle track in a bubble chamber, i.e. a number of aligned points together with some random points. The problem is to find the parameters **p** and **q** of a straight line

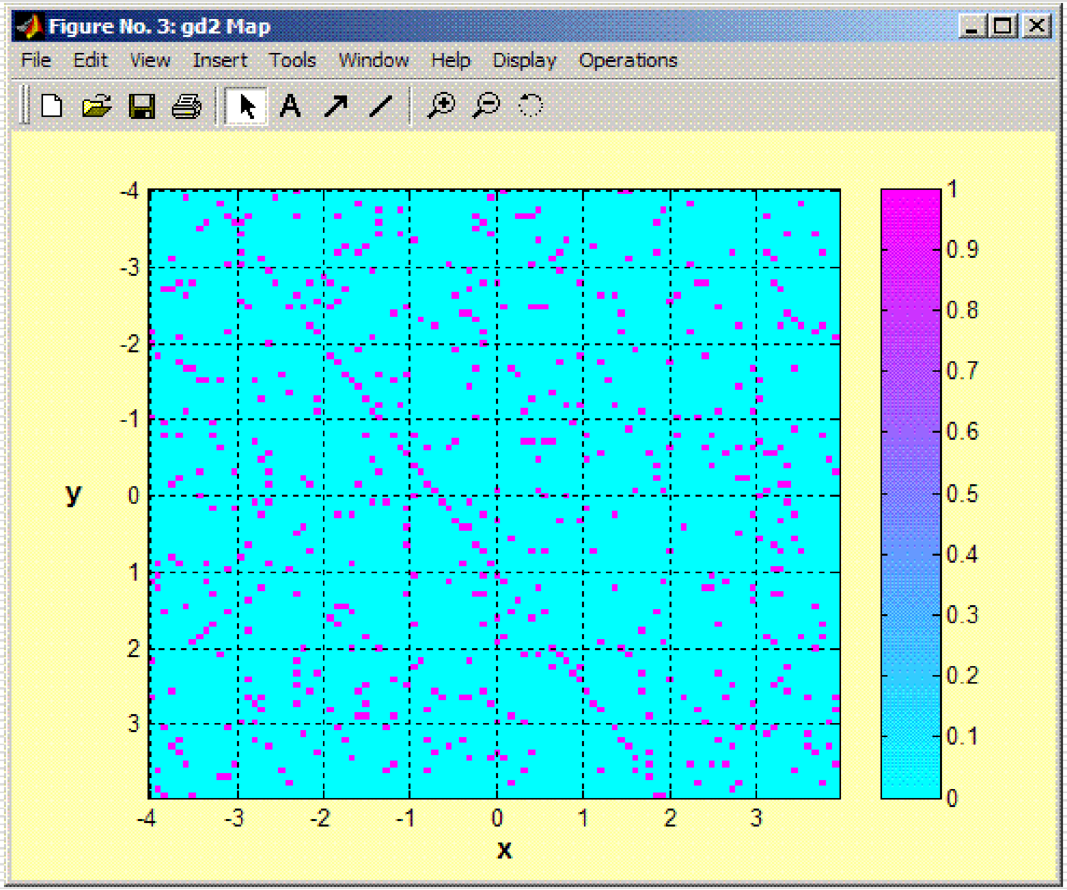
$$y = p * x + q$$

The "Hough transform" transform each point in the plane **(x,y)** to a straight line

$$q = - x * p + y$$

in the plane **(p,q)** and conversely a straight line in the **(x,y)** plane to a point in the **(p,q)** plane: the coordinate of the point in this plane are the parameters of the straight line.

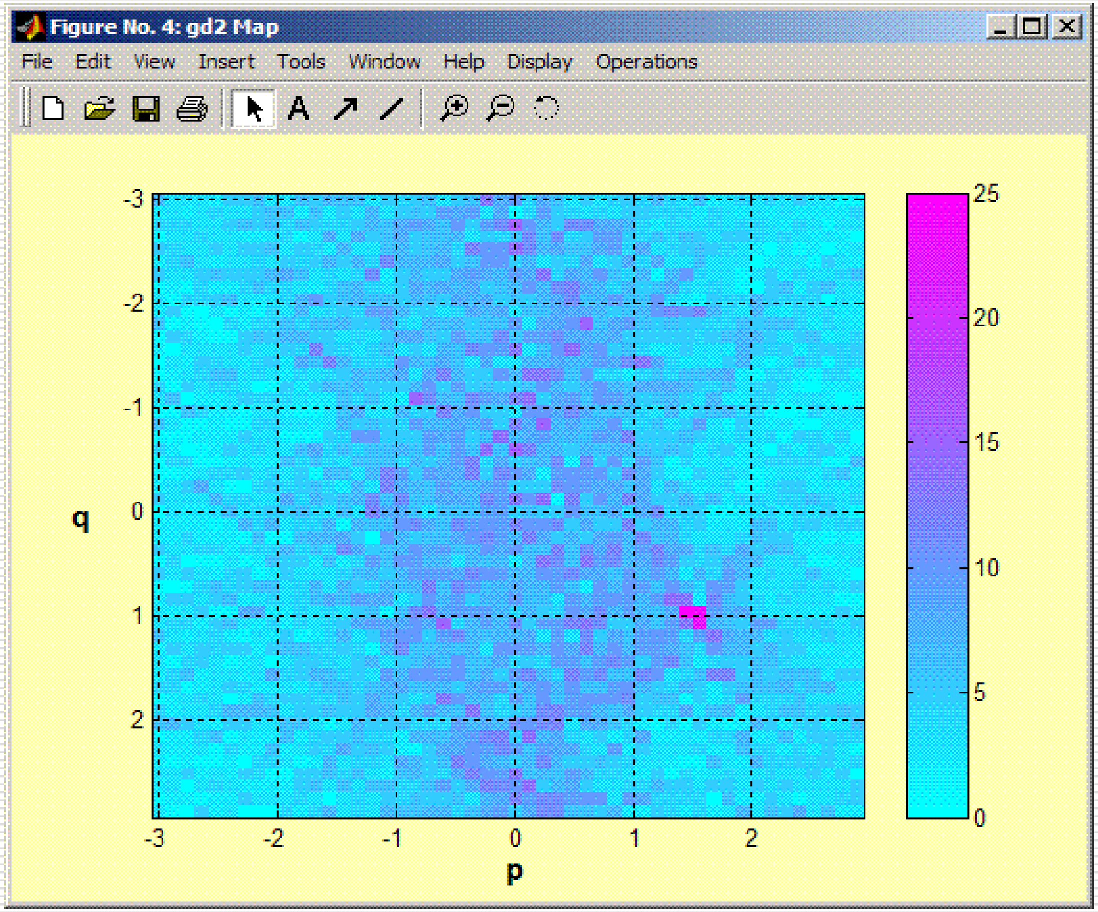
Peak Map - 1



Peak map (or bubble chamber image) with a straight line with equation

$$y = 1.5 * x + 1$$

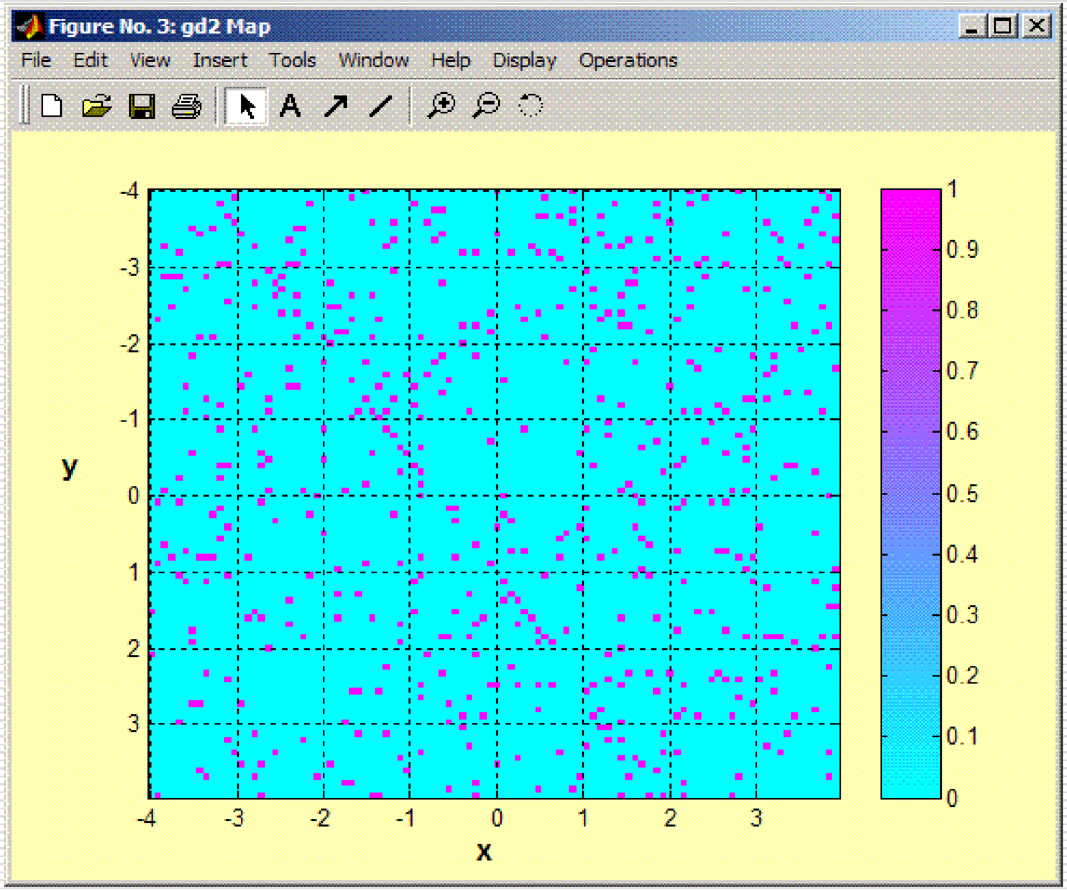
Hough Map - 1



Hough map of the preceding image. This can be seen as a 2-dimension histogram: for each point in the peak map, a set of aligned bins representing a straight line in the Hough map is increased by 1.

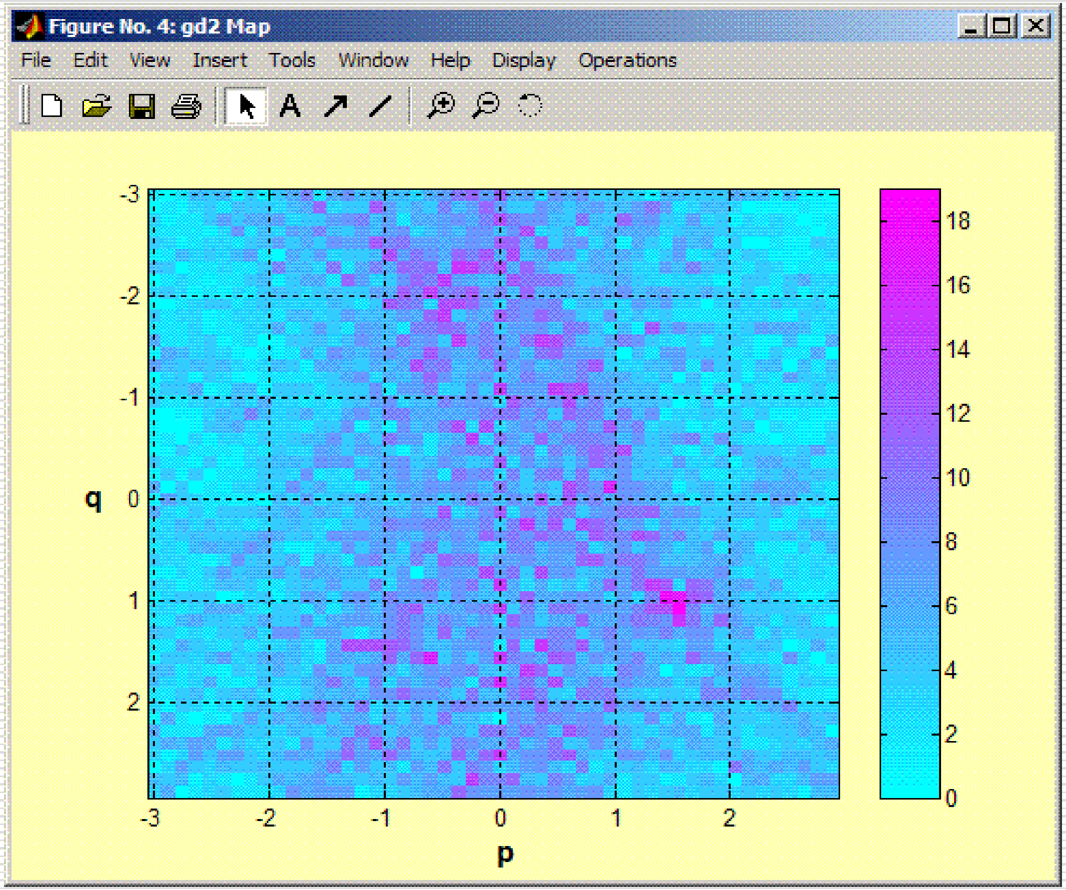
Note the peak at about p = 1.5 and q = 1

Peak Map - 2



The same of the preceding peak map, but with lower SNR (signal-to-noise ratio)

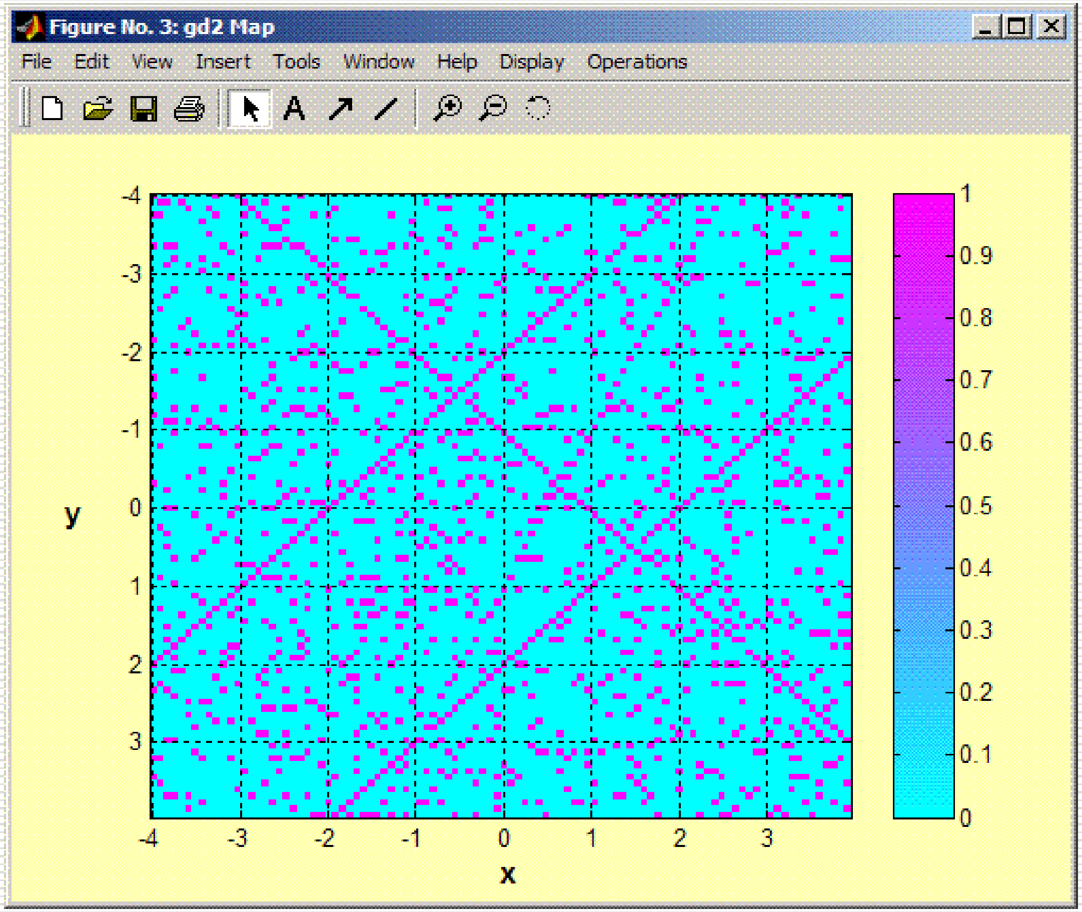
Hough map - 2



More noisy Hough map. The peak is always present, but there are also others, spurious.

Note that the noise is not uniform on the whole map.

Peak Map - 3



Peak map with 4 straight lines:

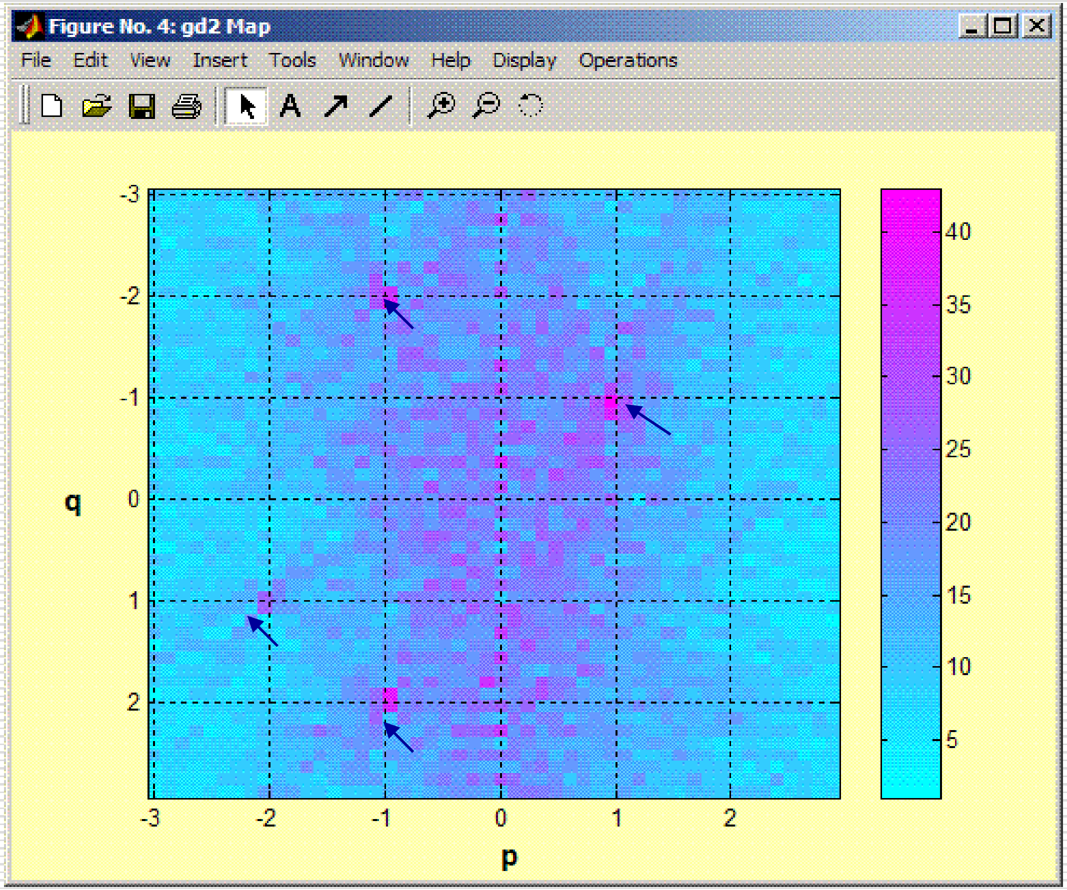
$$y = -x + 2$$

$$y = -x - 2$$

$$y = x - 1$$

$$y = -2 * x + 1$$

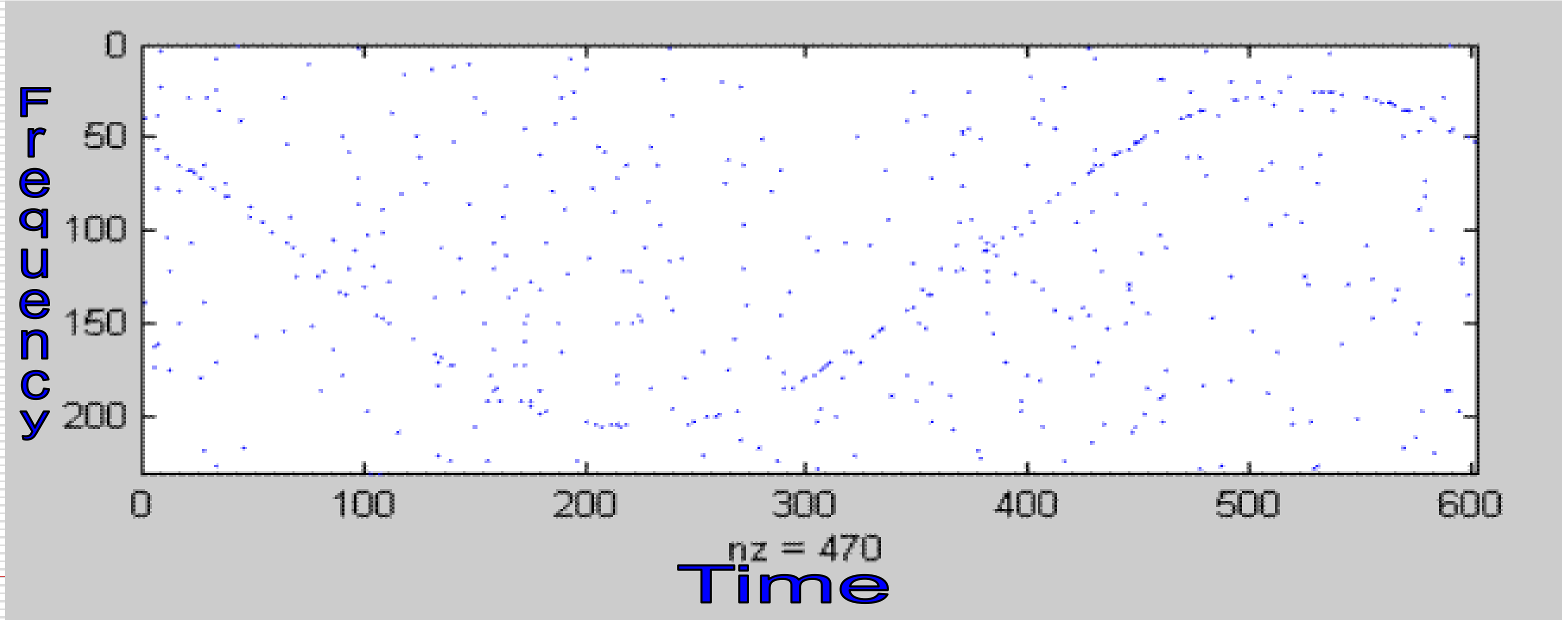
Hough map - 3



All the 4 straight line have been detected, with correct parameters.

Time-frequency peak map

Using the SFDB, create the periodograms and then a time-frequency map of the peaks above a threshold (about one year observation time).
Note the Doppler shift pattern and the spurious peaks.



Hierarchical method

- Divide the data in (interlaced) chunks; the length is such that the signal remains inside one frequency bin
- Do the FFT of the chunks; this is the SFDB
- Do the first "incoherent step" (Hough or Radon transform) and take candidates to follow
- Do the first "coherent step", following up candidates with longer "corrected" FFTs, obtaining a refined SFDB (on the fly)
- Repeat the preceding two step, until we arrive at the full resolution

Coherent steps

With the coherent step we partially correct the frequency shift due to the Doppler effect and to the spin-down. Then we can do longer FFTs, and so we can have a more refined time-frequency map.

This steps is done only on “candidate sources”, survived to the preceding incoherent step.

Coherent follow-up

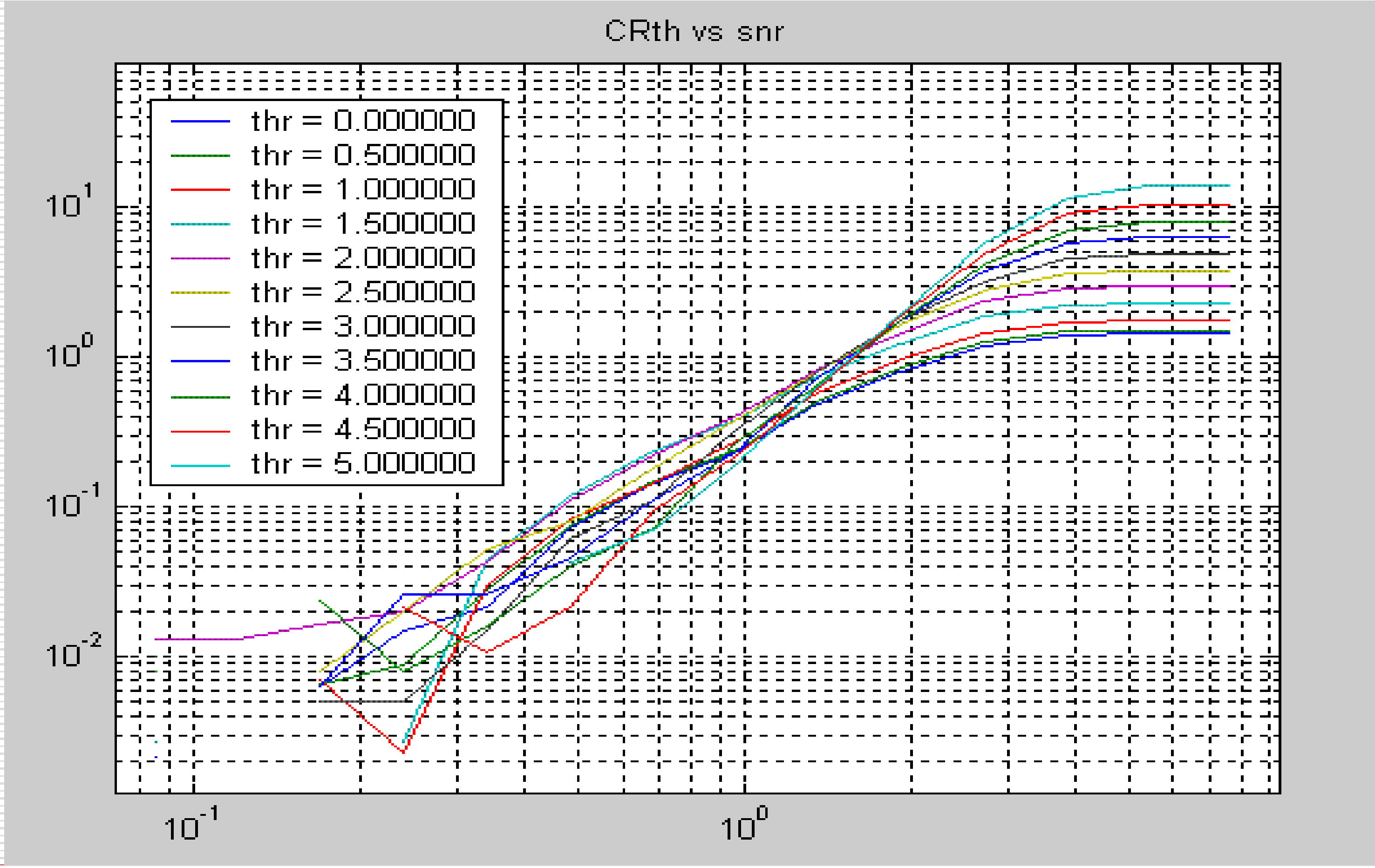
- Extract the band containing the candidate frequency (with a width of the maximum Doppler effect plus the possible intrinsic frequency shift)
- Obtain the time-domain analytic signal for this band (it is a complex time series with low sampling time (lower than 1 Hz)
- Multiply the analytic signal samples for $e^{-j\Delta\omega_D t_i}$, where t_i is the time of the sample, and $\Delta\omega_D$ is the correction of the Doppler shift and of the spin-down.
- Create a new (partial) FFT data base now with higher length (dependent on the precision of the correction) and the relative time-frequency spectrum and peak map
- Do the Hough transform on this (new incoherent step).

Hierarchical search results

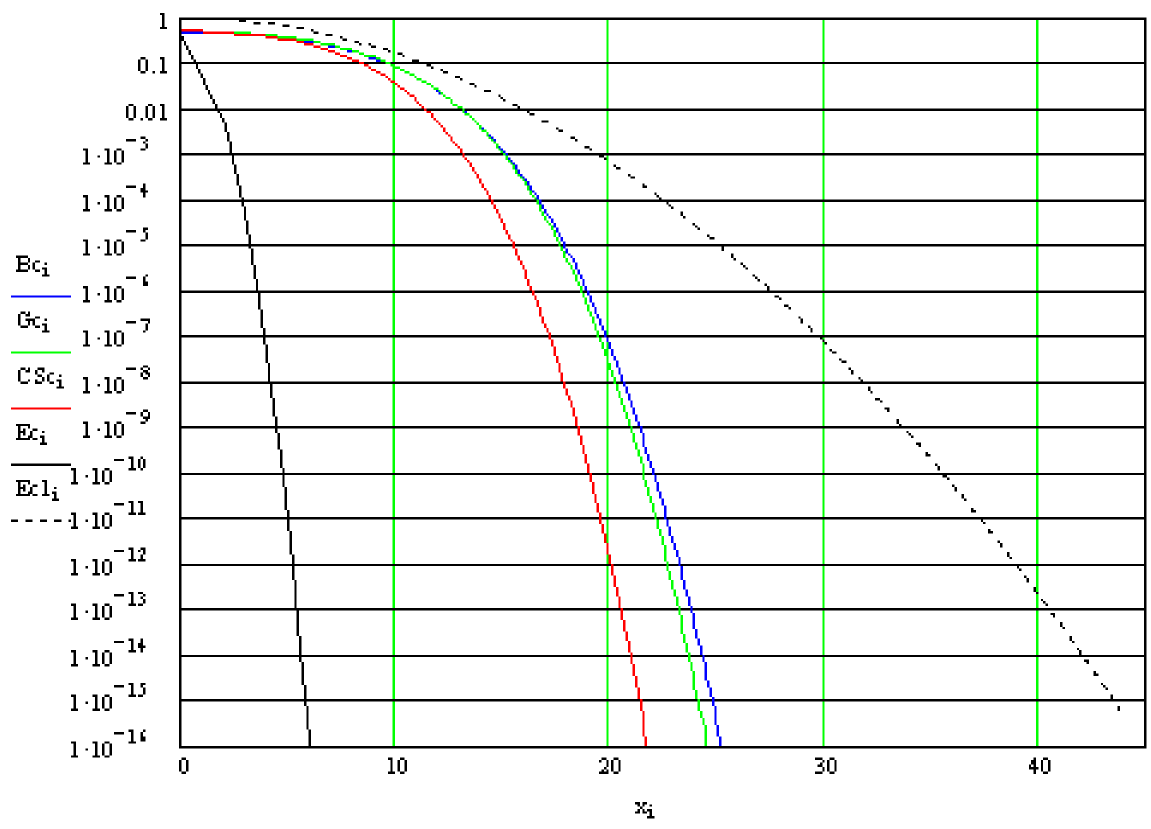
SFDB band	Band 1	Band 2	Band 3	Band 4
Doppler bandwidth (Hz)	0.2	0.05	0.0125	0.0032
Angular resolution in the sky (rad)	0.0038	0.0038	7.6294E-03	1.5259E-02
Number of pixels in the sky	8.6355E+05	8.6355E+05	2.1589E+05	5.3972E+04
Number of independent frequencies	1.5729E+06	1.5729E+06	7.8643E+05	3.9322E+05
Spin down parameters (only order 1)	140	140	70	35
Tot. number of parameters (one freq)	1.207E8	1.207E8	1.509E7	1.886E6
Number of operations for one peak	6.5884E+03	6.5884E+03	3.2942E+03	1.6471E+03
Total number of operations	6.348E+18	1.587E+18	4.959E+16	1.55E+15
Comp. Pow. for the 1st step (GFlops)	1030	257	8.0	0.251
Overall computing power (Gflops)	2000	500	15	0.5
Nominal sensitivity	6.17E-26	4.36E-26	3.67E-26	3.08E-26
Practical sensitivity	1.23E-25	8.72E-26	7.33E-26	6.17E-26

Minimum decay time considered is 10^4 years

Hough transform vs SNR



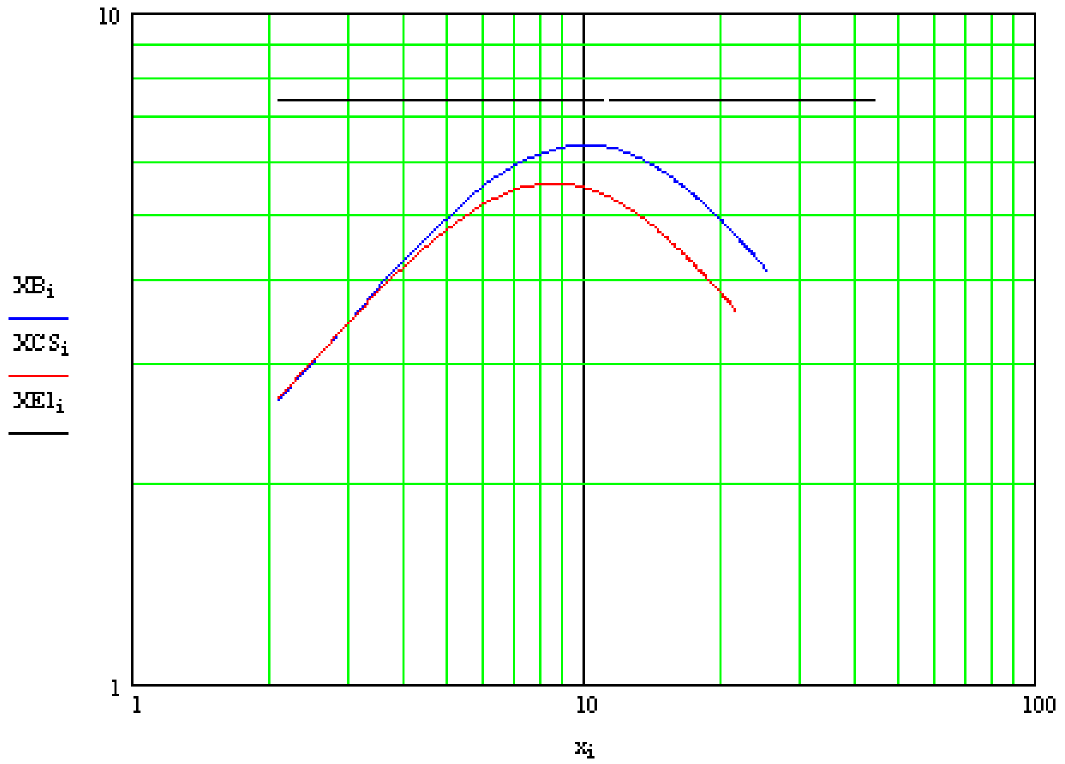
Noise distributions - linear



The black line is the noise distribution for the optimum detection, the red one is for the hierarchical procedure (hp) with Radon, the blue and green are for hp with Hough (the green is the gaussian approximation) and the dotted line is for a short FFT.

There were 3000 pieces.

Loss respect to the optimum



In this plot there is the SNR loss (respect to the optimum detectipon) for the hierarchical procedure with Hough (blue) and Radon (red) and a short FFT (black).

In abscissa there is the SNR..

Tuning a hierarchical search

The fundamental points are:

• the sensitivity is proportional to $\sqrt[4]{T_{FFT}}$

• the computing power for the incoherent step is proportional to T_{FFT}^3

• the computing power for the coherent step is proportional to $\log T_{FFT}$, but it is also proportional to the number of candidates that we let to survive.

What is a candidate source ?

The result of an analysis is a list of candidates (for example, 10^6 candidates).

Each candidate has a set of parameters:

- the frequency at a certain epoch
- the position in the sky
- 2~3 spin-down parameters

Detecting periodic sources

The main point is that a periodic source is permanent. So one can check the "reality" of a source candidate with the same antenna (or with another of comparable sensitivity) just doing other observations.

So we search for "coincidences" between candidates in different periods.

The probability to have by chance a coincidence between two sets of candidates in two 4-months periods is of the order of 10^{-20}.

Coincidences

In case of non-ideal noise, the preceding f.a. probabilities can be not reliable, nevertheless there are some methods to validate the survived candidates. One is the coincidence method.

If $\mathbf{n_1}$ and $\mathbf{n_2}$ candidates survive in two different four-months periods (for example $\mathbf{n_1 = n_2 = 10}$, at the third step, where the number of points in the parameter space $\mathbf{N_P}$ is about **6.e24**) , we can seek for coincidences between the two sets, i.e. check if there are some with equal (or similar) parameters.

The expected number of coincidences (or the probability of a coincidence) is

$$n_{COIN} = \frac{n_1 \cdot n_2}{N_P}$$

with the values of our example, $\mathbf{n_{COIN} = 6.e\text{-}22}$ **.**

False alarm probability

In the case of the periodic source search with the hierarchical method, the false alarm probability is normally embarrassingly low. This for two reasons:

- the hierarchical procedure produces at the first step a high number of candidates and for them the f.a. probability is practically 1, but already at the second step the candidates disappear and it plunges at very low levels.

- if some false candidates survive, the coincidence with the survived candidates (with the same parameters) in other periods or in other antennas lower the f.a. probability at levels of absolute impossibility.

Computing Hough f.a. probability

Let us start from a random peak map. Let **p** (~0.1) be the density of the peaks on the map. The value k of a pixel of the Hough map follows a binomial distribution

$$\binom{M}{k} p^k (1-p)^{M-k}$$

where **M** is the number of spectra.

If there is a weak signal, the expected value of k is enhanced by an amount proportional to the square of the amplitude of the signal. So if there is a certain (linear) **SNR** at a certain step, at the following one, with a 16 times longer $\mathbf{T_{FFT}}$, there is a **CR** four times higher.

Sensitivity

The signal detectable with a CR of 4 (5.E10 candidates in the band from 156 to 625 Hz) is given by

$$h_{CR=4} \approx 2 \frac{2S_h}{\sqrt[4]{T_{OBS} \cdot T_{FFT}}} = 2.8 \cdot 10^{-25}$$

with T_{OBS}=4 months, T_{FFT}=3355 s, S_h=3E-23 $Hz^{-1/2}$.